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Noveno Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemática, Pontificia Universidad Católica de Chile
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Directional $\mathcal{H}^2$-matrices for Helmholtz integral operators

Steffen Börm* Jens M. Melenk†

Abstract

Boundary Element Methods (BEM) are an important tool for the numerical solution of acoustic and electromagnetic scattering problems. These BEM matrices are fully populated so that data-sparse approximations are required to reduce the complexity from quadratic to log-linear. For the high-frequency case of large wavenumber, standard blockwise low-rank approaches are insufficient. One possible data-sparse matrix format for this problem class that can lead to log-linear complexity are *directional* $\mathcal{H}^2$-matrices. [1, 2, 4, 9]. We present a full analysis of a specific incarnation of this approach, [4]. Directional $\mathcal{H}^2$-matrices are blockwise low rank matrices, where the block structure is determined by the so-called parabolic admissibility condition, [6]. In order to achieve log-linear complexity with this admissibility condition, a nested multilevel structure such as $\mathcal{H}^2$-matrices [7] is essential, which provides a data-sparse connection between clusters of source and target points on different levels. We present a particular variant of directional $\mathcal{H}^2$ matrices in which all pertinent objects are obtained by polynomial interpolation. This allows us to rigorously establish exponential convergence in the block rank in conjunction with log-linear complexity. We will also discuss the relation of the directional $\mathcal{H}^2$-matrices to Butterfly Algorithms, [8, 5, 3].

Key words: Helmholtz equation, boundary element method, matrix compression, multipole method

Mathematics subject classifications (1991): 35J05, 65D05, 65N38, 41A10, 65N12

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*Institut für Informatik, University of Kiel, Kiel, Germany, email: sb@informatik.uni-kiel.de.
†Institute for Analysis and Scientific Computing, Technische Universität Wien, Vienna, Austria, email: melenk@tuwien.ac.at

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