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A multi-class model for batch settling in WRRFs *

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Abstract

In order to achieve a unified description of the settling processes in water resource recovery facilities (WRRFs) taking place in both primary settling tanks (PSTs) and secondary settling tanks (SSTs) in conventional wastewater treatment, a new framework, based on the state of the art Bürger-Diehl settling model for SSTs [2], was introduced in [4]. This new unified framework is built on the idea that the distributed properties of the sludge can be captured by dividing the total sludge concentration into a number of classes, depending on the settling velocity distribution. From the mathematical point of view, the extension to a multi-class scenario leads us to a system of nonlinear convection-diffusion equations of the type

$$\frac{\partial \boldsymbol{X}}{\partial t} + \frac{\partial \boldsymbol{f}(\boldsymbol{X})}{\partial z} = \frac{\partial}{\partial z}\left(\boldsymbol{B}(\boldsymbol{X})\frac{\partial \boldsymbol{X}}{\partial z}\right), \tag{1}$$

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where $\boldsymbol{X} = (X_1, \dots, X_N)^{\mathrm{T}}$ is the sought solution depending on the spatial position z and time t, $X_i = X_i(z,t)$ the mass concentration of class i, $i = 1, \dots, N$, where N is the number of classes considered, $\boldsymbol{f}(\boldsymbol{X}) = (f_1(\boldsymbol{X}), \dots, f_N(\boldsymbol{X}))$ is a vector of convective flux density functions modelling the settling of the sludge and $\boldsymbol{B}(\boldsymbol{X})$ is a given $N \times N$ matrix expressing the diffusive correction, in this case, due to the solids compressibility. This system has to be supplied with initial and boundary conditions. It is well known that under the typical assumptions of sedimentation with compression, (1) is a strongly degenerate parabolic system, while when settling effects are dominant, and $\boldsymbol{B}(\boldsymbol{X}) = \boldsymbol{0}$, it is a first-order, nonlinear hyperbolic system of conservation laws. Due to the nonlinearity of $\boldsymbol{f}$ as a function of $\boldsymbol{X}$ in combination with the degenerate behaviour, discontinuities or sharp gradients are expected to develop. This property calls for specific techniques for the numerical simulations. The use of implicit-explicit Runge-Kutta (IMEX-RK) schemes [1], along with the weighted essentially non-oscillatory (WENO) shock-capturing technology for the discretization of the set of equations (1), is advocated in [3]. These schemes combine an explicit treatment for the time discretization of the convective terms with an implicit treatment of the diffusive ones, with the result that the resulting IMEX scheme enjoys a less restrictive stability condition than a fully explicit scheme. The use of high resolution shock-capturing finite difference WENO schemes for the discretization of the convective term ensure obtaining precise numerical approximations, accurately resolving the shocks arising and avoiding the spurious oscillations that otherwise often appear.

Key words: multi-class kinematic flow model, wastewater treatment, convection-diffusion equation, primary settling tank, secondary settling tank, settling velocity distribution

Mathematics subject classifications (1991): 35L65, 65M06, 76T05

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