

Variational formulation of time-fractional parabolic equations*

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Abstract

We consider initial/boundary value problems for time-fractional parabolic PDE of order $1/2 < \alpha < 1$, that is, $\partial_t^\alpha u - \Delta u = f$ where ∂_t^α is a fractional time-derivative. Equations of this kind model diffusion phenomena where the mean-square displacement of a diffusing particle scales non-linear in time (as opposed to e.g., the well-known Brownian motion). Recently, researchers have started to analyze finite element methods with respect to their ability to approximate solutions of fractional PDE. In our talk, based on the work [1], we present a variational formulation of time-fractional parabolic equations which resembles classical results for parabolic PDE. This includes the extension of operators defined on real-valued Sobolev spaces to their Banach space-valued counterparts, the so-called *Sobolev-Bochner spaces*, as well as Sobolev Embedding results. This way, we provide a theoretical underpinning for the numerical analysis of such equations.

Key words: Fractional diffusion, Initial/boundary value problem, Well-posedness

Mathematics subject classifications (1991): 26A33, 35K15, 35R11

References

- [1] M. KARKULIK, *Variational formulation of time-fractional parabolic equations*. arXiv:1704.03257, submitted.

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