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Noveno Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemática, Pontificia Universidad Católica de Chile
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Multiple traces formulation and semi-implicit scheme for modeling packed cells under electrical stimulation*

Fernando Henríquez† Carlos Jerez-Hanckes‡

Abstract

We model the electrical behavior of several biological cells under external stimuli by extending and computationally improving the semi-implicit multiple traces formulation presented in [3]. Therein, the electric potential and current for a single cell are retrieved through the coupling of boundary integral operators and non-linear ordinary differential systems of equations. Yet, the low-order discretization scheme presented becomes impractical when accounting for interactions among multiple cells. In this note, we consider multi-cellular systems and show existence and uniqueness of the resulting non-linear evolution problem in finite time. Our main tools are analytic semigroup theory along with mapping properties of boundary integral operators. Thanks to the smoothness of cellular shapes, solutions are highly regular at a given time. Hence, spectral spatial discretization can be employed, thereby largely reducing the number of unknowns. Time-space coupling is achieved via a semi-implicit time-stepping scheme shown to be stable and convergent. Numerical results in two dimensions validate our claims and match observed biological behavior for the Hodgin-Huxley dynamical model.

Key words: semi-implicit method, multiple traces formulation, hodgin-huxley model, spectral method

Mathematics subject classifications (2000): 65M38, 65M12, 65R20

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†Seminar for Applied Mathematics, ETH Zurich, Raemistrasse 101, CH8092, Zurich, Switzerland, email: fernando.henriquez@sam.math.ethz.ch.

‡Institute of Mathematical & Computational Engineering, School of Engineering, Pontificia Universidad Católica de Chile, Vicuña Mackenna 4860, 7820436, Santiago, Chile, email: cjerez@ing.puc.cl

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