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A mixed VEM scheme for a problem with edge and vertex singularities

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Abstract

We introduce and analyze a virtual element method [4] for the mixed formulation of a Poisson problem with right–hand side in L^2 and homogeneous Dirichlet conditions in a non-convex polyhedral domain with edge and vertex singularities, for which, in the presence of the mentioned singularities, it is known that its solution in general is not in H^2 (cfr. [2, 3]). As a consequence, the usual Finite Elements Methods are degraded and we do not obtain an optimal convergence order in the general case. We present a VEM constructing a mesh that combines anisotropic prisms and tetrahedra with pyramids and avoids the use of certain tetrahedra that do not admit anisotropic estimates, recovering the optimal order of convergence. As stated in [1], if we make a subdivision of a general polyhedron Ω only with tetrahedra, then we do not obtain optimal error estimates with Mixed Raviart–Thomas Finite Elements for our problem. That is because there exists a class of anisotropic tetrahedra for which anisotropic estimates needed in the analysis do not hold. For that reason we propose a method which among other things avoids the use of that kind of tetrahedra. In order to deal with general polyhedral domains we need to use mixed meshes, so we present a VEM scheme in a polyhedral mesh $\mathcal{T}_h$ made of tetrahedra, triangular prisms and pyramids. This scheme can be seen as an extension of the method with classical lowest order Raviart–Thomas elements to the case in which the mesh contains pyramids. Besides, it is also an alternative to the generalization of the $H(\mathrm{div})$–conforming elements on pyramids found for instance in [5], whose spaces, in particular, contain rational functions. Incidentally, the number of mesh elements in our method is reduced by a constant factor. We show a discretization method and introduce the corresponding discrete bilinear forms and show that the discrete problem is well posed by proving the discrete $\inf - \sup$ condition. Next we prove that there exists a family of graded meshes $\{\mathcal{T}_h\}_{h\downarrow 0}$ for which we have the optimal estimation $\|\mathbf{u} - \mathbf{u}_h\| \leqslant ch\|f\|$, $\|p - p_h\| \leqslant ch\|f\|$, with $h \lesssim (1/N)^{1/3}$, where N is the number of elements of the mesh $\mathcal{T}_h$. We show an example of a family of meshes for the Fichera domain that verifies our hipothesis.

Key words: virtual element method, mixed formulation of Poisson Problem, a priori error analysis

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