

A fully discrete scheme for the pressure-stress formulation of a time-domain fluid-structure interaction problem*

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Abstract

We propose an implicit Newmark method for the time integration of the pressure-stress formulation of a fluid-structure interaction problem. The space Galerkin discretization is based on the Arnold-Falk-Winther mixed finite element method with weak symmetry in the solid and the usual Lagrange finite element method in the acoustic medium. We prove that the resulting fully discrete scheme is well-posed and uniformly stable with respect to the discretization parameters and Poisson ratio, and we provide asymptotic error estimates. Finally, we present numerical tests to confirm the asymptotic error estimates predicted by the theory.

Key words: mixed finite elements, fluid-solid interaction, error estimates

Mathematics subject classifications (1991): 65N30, 65M12, 65M15, 74H15

References

- [1] D. N. ARNOLD, R. S. FALK, AND R. WINTHER, *Mixed finite element methods for linear elasticity with weakly imposed symmetry*. Math. Comp. 76 (2007), no. 260, 1699–1723.
- [2] A. BERMÚDEZ, R. RODRÍGUEZ, AND D. SANTAMARINA, *Finite element approximation of a displacement formulation for time-domain elastoacoustic vibrations*. J. Comput. Appl. Math. 152 (2003), no. 1-2, 17–34.

*This work was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile; and by Centro de Investigación en Ingeniería Matemática (CI²MA), Universidad de Concepción.

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- [3] D. BOFFI, F. BREZZI, AND M. FORTIN, *Mixed Finite Element Methods and Applications*. Springer Series in Computational Mathematics, 44. Springer, Heidelberg, 2013.
- [4] X. FENG, *Analysis of finite element methods and domain decomposition algorithms for a fluid-solid interaction problem*. SIAM J. Numer. Anal. 38 (2000), no. 4, 1312–1336.
- [5] C. GARCÍA, G.N. GATICA, AND S. MEDDAHI, *A new mixed finite element analysis of the elastodynamic equations*. Appl. Math. Lett. 59 (2016), 48–55.
- [6] C. GARCÍA, G.N. GATICA, AND S. MEDDAHI, *A new mixed finite element method for elastodynamics with weak symmetry*. J. Sci. Comput., to appear.
- [7] C. GARCÍA, G.N. GATICA, AND S. MEDDAHI, *Finite element analysis of a pressure-stress formulation for the time-domain fluid-structure interaction problem*. IMA J. Numer. Anal., to appear.
- [8] G.N. GATICA, *A Simple Introduction to the Mixed Finite Element Method. Theory and Applications*. SpringerBriefs in Mathematics. Springer, Cham, 201
- [9] G.N. GATICA, A. MÁRQUEZ, AND S. MEDDAHI, *Analysis of the coupling of Lagrange and Arnold-Falk-Winther finite elements for a fluid-solid interaction problem in 3D*. SIAM J. Numer. Anal. 50 (2012), no. 3, 1648–1674.