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A posteriori error analyses for augmented mixed formulations of the Boussinesq model*

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Abstract

In previous works of us, new augmented mixed finite element schemes were developed for the stationary Boussinesq problem describing heat driven flow. Our methodologies consisted of a fixed-point strategy for the variational problems that resulted after introducing the same modified pseudostress tensor as an auxiliary unknown in the Navier-Stokes type system involved in the model and, separately, the normal component of the temperature gradient and a vector depending on the temperature, its gradient and the fluid velocity as auxiliary variables in the advection-diffusion equation describing the heat transfer. In both cases, suitable parameterized redundant Galerkin terms were incorporated to the schemes. The well-posedness of both the continuous and discrete settings, the convergence of the associated Galerkin schemes, as well as a priori error estimates of optimal order were stated there. In this talk we present the corresponding a posteriori error analyses of our aforementioned augmented methods in two and three dimensions. Standard arguments relying on duality techniques, and suitable Helmholtz decompositions are used to derive global error indicators and to show their reliability. A globally efficiency property with respect to the natural norm for each estimator is further proved via usual localization techniques of bubble functions. Finally, adaptive algorithms based on reliable, fully local and computable a posteriori error estimators are proposed, and their performance and effectiveness are illustrated through a few numerical examples in two dimensions.

Key words: Boussinesq model, augmented mixed formulation, a posteriori error analysis, reliability, efficiency, adaptive algorithm.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 76D07

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