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Analysis of an augmented fully-mixed formulation for the non-isothermal Oldroyd–Stokes problem*

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Abstract

In this work we present an augmented mixed finite element method for the Oldroyd–Stokes problem describing the motion of a non-isothermal incompressible fluid subject to a heat source. The model is described by a system of equations where the Stokes and heat equations are coupled through the convective term and the viscosity of the fluid. We introduce the strain, stress and vorticity tensors, as well as the gradient of the temperature, as further unknowns, which together with the velocity, and the temperature of the fluid, constitute the main unknowns of the system. The pressure is eliminated from the system and can be recovered through a simple post-process of the solution. Since the convective term in the heat equation forces both the velocity and the temperature to live in a smaller space than usual, we augment the variational formulation by using the constitutive and equilibrium equations, the relation defining the strain and vorticity tensors, and the temperature boundary condition. Next, we combine the well-known Schauder and Banach fixed-point theorems with the Lax–Milgram lemma and prove existence and uniqueness of solution of the resulting augmented fully-mixed formulation. The associated Galerkin scheme is defined by Raviart–Thomas elements of order k for the stress tensor and the heat flux vector, continuous piecewise polynomials of order $< k+1$ for velocity and temperature, and piecewise polynomials of order $< k$ for the strain tensor and the vorticity, and its solvability is established similarly to its continuous counterpart, using in this case Brouwer fixed-point theorem for the existence of solution. Finally, we derive optimal a priori error estimates and provide several numerical results illustrating the good performance of the scheme and confirming the theoretical rates of convergence.

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