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An augmented stress-based mixed finite element method for the Navier-Stokes equations with nonlinear viscosity*

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Abstract

A new stress-based mixed variational formulation for the Navier-Stokes equations with constant density and variable viscosity depending on the magnitude of the strain tensor, is proposed and analyzed in this work. Our approach is a natural extension of a technique applied in a recent paper by some of the authors to the same boundary value problem but with a viscosity that depends nonlinearly on the gradient of velocity instead of the strain tensor. In the present case, and besides remarking that the strain-dependence for the viscosity yields a physically more meaningful model, we notice that in order to handle this nonlinearity we now need to incorporate not only the strain itself but also the vorticity as auxiliary unknowns. Furthermore, similarly as in that previous work, and aiming to deal with a suitable space for the velocity, the variational formulation is augmented with Galerkin type terms arising from the constitutive and equilibrium equations, the relations defining the two additional unknowns, and the Dirichlet boundary condition. In this way, and since the resulting augmented scheme can be rewritten as a fixed point operator equation, the classical Schauder and Banach theorems together with monotone operators theory are applied to derive the well-posedness of the continuous and associated discrete schemes. In particular, we show that arbitrary finite element subspaces can be utilized for the latter, and then we derive

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optimal a priori error estimates and the corresponding rates of convergence. Next, a reliable and efficient residual-based a posteriori error estimator on arbitrary polygonal and polyhedral regions is proposed. The main tools employed include Raviart-Thomas and Clément interpolation operators, inverse and discrete inequalities, and the localization technique based on triangle-bubble and edge-bubble functions. Finally, several numerical essays illustrating the good performance of the method, confirming the reliability and efficiency of the a posteriori error estimator, and showing the desired behaviour of the adaptive algorithm, are reported.

Key words: Navier-Stokes equations, nonlinear viscosity, augmented mixed formulation, fixed point theory, mixed finite element methods, a priori error analysis

Mathematics subject classifications (2000): 65N30, 65N12, 65N15, 35Q79, 80A20, 76R05, 76D07

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