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Departamento de Matemática, Pontificia Universidad Católica de Chile
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A finite volume method for 3D convective solidification *

Roberto C. Cabrales† Francisco Guillén-González‡
Juan Jaime§ Nelson O. Moraga§

Abstract

Solidification is a phase change problem appearing in several engineering applications such as liquid to solid phase transformations of metals and alloys, food freezing and freeze-drying for pharmaceutical drugs production. The mathematical model (see [1]) for this problem is formed by a system of nonlinear partial differential equations jointly with auxiliary expressions relating the variables of the problem: the velocity of the fluid $\boldsymbol{u}$, the pressure p, the enthalpy H, the temperature T and the phase change function f_{pc}. The last variable depends on T and describes the state of every point on the calculation domain Ω: if T is lower than the solid temperature T_s, $f_{pc}(T) = 0$ and we are in solid phase; if T is greater than the liquid temperature T_l, $f_{pc}(T) = 1$ and we are in liquid phase, if $T_s < T < T_l$, $f_{pc}(T) = (T_l - T_s)^{-1}(T - T_s)$, and we have coexistence of both phases. Additionally, it is assumed that the only external force acting on the system is the gravity $\boldsymbol{g}$. The numerical method is based on the finite volume method. Each variable is calculated in a sequential form and the velocity-pressure coupling is managed by using a SIMPLE-like algorithm developed in [2]. To verify the computational implementation of the algorithm, we consider 3D natural convection problem for several Rayleigh numbers from 10^3 to 10^6 and compare with those presented in [3]. The investigated case is the solidification of a Al-Si alloy in a 3D cavity with a graphite mold, extending the 2D results of [4].

Key words: finite volume method, SIMPLE algorithm, 3D convective solidification

Mathematics subject classifications (2010): 65M08, 80A22, 76M12

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†Instituto de Investigación Multidisciplinario en Ciencia y Tecnología, Universidad de la Serena, Benavente 980, La Serena, Chile. email: `rcabrales@userena.cl`.

‡Dpto. EDAN & IMUS, Universidad de Sevilla, Aptdo. 1160, 41080, Sevilla, España, email: `guillen@us.es`.

§Departamento de Ingeniería Mecánica, Universidad de La Serena, Casilla 445, La Serena, Chile. email: `jjaime@userena.cl, nmoraga@userena.cl`.

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