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The upwind hybridized discontinuous Galerkin (HDG) framework: Theory and application to magnetohydrodynamic and atmospheric applications

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Abstract

By revisiting the classical Godunov approach for linear system of hyperbolic Partial Differential Equations (PDEs) we show that it is hybridizable. As such, it is a natural recipe for us to constructively and systematically establish a unified HDG framework for a large class of PDEs including those of Friedrichs type. The unification is fourfold. First, it provides a single constructive procedure to devise HDG schemes for elliptic, parabolic, hyperbolic, and mixed-type PDEs. Second, it reveals the nature of the trace unknowns as the Riemann solutions. Third, it provides a parameter free HDG framework, and hence eliminating the usual complaint that HDG is a parameter-dependent method. Fourth, it allows us to construct the existing HDG methods in a systematic manner. In particular, using the unified framework we can rediscover most of the existing HDG methods and furthermore discover new ones. We present a rigorous wellposedness of the upwind HDG framework for abstract PDEs of Friedrichs' type. Convergent analysis will be established for PDEs arising from Magnetohydrodynamic and atmospheric applications. For nonlinear PDEs, we present an IMEX scheme that exploits the HDG method to solve a single small linear system in each time step: a tremendous advantage over traditional approaches. Part of the talk are the multilevel HDG and iterative HDG approaches that we have developed to solve the HDG systems efficiently on parallel supercomputers. Serial and parallel numerical results for various PDEs will be presented to verify and demonstrate our upwind HDG framework.

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