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An adaptive Galerkin method for the time-dependent complex Schrödinger equation *

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Abstract

Nonlinear time-dependent Schrödinger equations (NLSE) model several important problems in quantum physics and morphogenesis. Recently, vortex lattice formation were experimentally found in Bose-Einstein condensate and Fermi superfluids, which are modeled by adding a rotational term in the NLSE equation. Numerical solutions have been computed by using separate approaches for time and space variables. If we see the complex equation as a system, wave methods can be used. In this article, we consider finite element approximations using continuous Galerkin schemes in time and space by adaptive mesh balancing both errors. To get this balance, we adapt the dual weighted residual method used for wave equations and estimates of error indicators for adaptive space-time finite element discretization. The results show how important is dynamic refinement to control the degrees of freedom in space.

Key words: nonlinear time-dependent Schrödinger equation, dual weighted residual method, adaptive Galerkin method

Mathematics subject classifications (2000): 35B40, 35P30, 35Q55, 65N25, 81Q05.

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