

A posteriori error analysis of a fully-mixed formulation for the Brinkman-Darcy problem*

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Abstract

We develop the *a posteriori* error analysis for a mixed finite element method applied to the coupling of Brinkman and Darcy equations in 3D, modelling the interaction of viscous and non-viscous flow effects across a given interface. The system is formulated in terms of velocity and pressure within the Darcy subdomain, together with vorticity, velocity and pressure of the fluid in the Brinkman region, and a Lagrange multiplier enforcing pressure continuity across the interface. The solvability of the fully-mixed formulation along with *a priori* error estimates for a finite element method have been recently established in [M. Alvarez et al., Comput. Methods Appl. Mech. Engrg. 307 (2016) 68–95]. Here we derive a residual-based *a posteriori* error estimator for such a scheme, and we prove its reliability exploiting a global inf-sup condition in combination with suitable Helmholtz decompositions, and properties of Clément and Raviart-Thomas operators. The estimator is also shown to be efficient, following a localisation strategy and appropriate inverse inequalities. We present some numerical tests to confirm the features of the estimator and to illustrate the performance of the method in a number of application-oriented problems.

Key words: Brinkman-Darcy equations, vorticity-based formulation, mixed finite element methods, *a posteriori* error analysis.

Mathematics subject classifications (1991): 65N30, 65N12, 76D07, 65N15

*This work was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile; and by Centro de Investigación en Ingeniería Matemática (CI²MA), Universidad de Concepción.

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