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The discrete relations between fields and potentials with high order Whitney forms

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Abstract

Besides the list of nodes and of their positions, the mesh data structure also contains incidence matrices, saying which node belongs to which oriented edge, which oriented edge bounds which oriented face and so on. These matrices contain all the information about the topology of the domain. Moreover, when using Whitney elements on simplices [2], they connect the dofs describing potentials to dofs describing fields. As an example, the relation $\mathbf{E} = -\text{grad}\, V$ between the electric field $\mathbf{E}$ and the scalar electric potential V become at the discrete level $\mathbf{e} = -G\mathbf{v}$ where G is the transpost of the node-to-edge incidence matrix and $\mathbf{e}$ and $\mathbf{v}$ are the vectors of edge circulations and values at nodes of $\mathbf{E}$ and V respectively. When fields and potentials are approximated by polynomial differential forms of higher degree, the discrete equivalent of the field/potential relation is more structured. The involved matrices present a structure by blocks, each block taking into account of the transmission of dofs associated to a geometrical dimension. We wish to investigate the block-structure of these matrices, when fields and potentials are approximated by high order Whitney forms [5], with dofs given either by the wellknown moments [4, 1] or by the more recent weights on the small simplices [3].

Key words: Discrete potentials, Whitney forms, inicidence matrices, high order approximations.

Mathematics subject classifications (1991): 78M10, 65N30, 68U20

References

[1] D. N. Arnold, R.S. Falk and R. Winther, *Finite element exterior calculus, homological techniques, and applications*. Acta Numerica 15 (2006), 1–155.

[2] A. Bossavit, *Computational Electromagnetism*. Academic Press, New York, 1998.

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[3] S. H Christiansen and F. Rapetti, *On high order finite element spaces of differential forms* Math. of Comp. 85 (2015), 517–548.

[4] J.-C Nédélec, *Mixed finite elements in* $\mathbf{R}^3$. Numer. Math. 35 (1980), 315–341.

[5] F. Rapetti and A. Bossavit, *Whitney forms of higher degree*. SIAM J. Numer. Anal. 47 (2009), 2369–2386.