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FE approximations of the nonhomogeneous fractional Dirichlet problem *

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Abstract

We study finite element approximations of the following non-homogeneous Dirichlet problem

$$\begin{cases} (-\Delta)^s u = f & \text{in } \Omega, \\ u = g & \text{in } \Omega^c, \end{cases} \tag{1}$$

on a bounded domain $\Omega \subset \mathbb{R}^n$. The operator $(-\Delta)^s$ stands for the Fractional Laplacian and the functions f and g belong to suitable spaces. Our approach is based on weak imposition of the Dirichlet condition and incorporating a nonlocal analogous of the normal derivative as a Lagrange multiplier in the formulation of the problem. In order to obtain convergence orders for our scheme, regularity estimates are developed, both for the solution and its nonlocal derivative. The method we propose requires that, as meshes are refined, the discrete problems be solved in a family of domains of growing diameter.

Key words: Fractional Laplacian, Mixed Methods, a priori error analysis

Mathematics subject classifications (1991): 65N30, 65N12, 35S15

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