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A mixed virtual element method for quasi-Newtonian Stokes flows*

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Abstract

In this paper we introduce and analyze a virtual element method (VEM) for an augmented mixed variational formulation of a class of nonlinear Stokes models arising in quasi-Newtonian fluids. While the original unknowns are given by the pseudostress, the velocity, and the pressure, the latter is eliminated by using the incompressibility condition, and in order to handle the nonlinearity involved, the velocity gradient is set as an auxiliary one. In this way, and adding a redundant term arising from the constitutive equation relating the psdeudostress and the velocity, an augmented formulation showing a saddle point structure is obtained, whose well-posedness has been established previously by using known results from nonlinear functional analysis. Then, following the basic principles and ideas of the mixed-VEM approach, we introduce a Galerkin scheme employing generic virtual element subspaces and projectors satisfying suitable abstract conditions, and derive the corresponding solvability analysis, along with the associated *a priori* error estimates for the virtual element solution as well as for the fully computable projection of it. Next, we provide two specific choices of subspaces and local projectors verifying the required hypotheses, one of them yielding an optimally convergent mixed-VEM for the fully nonlinear problem studied here, and the other one providing a new approach for the linear version of it, that is for the Stokes problem. In addition, we are able to apply a second element-by-element postprocessing formula for the pseudostress, which yields an optimally convergent approximation of it with respect to the broken $\mathbb{H}(\mathbf{div})$-norm. Finally, several numerical results illustrating the good performance of the method and confirming the theoretical rates of convergence are reported.

Key words: nonlinear Stokes equations, virtual element method, a priori error analysis

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 76D07

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