

# WINDOWED GREEN FUNCTION METHOD AND MAXWELL EIGENFUNCTIONS FOR OPEN WAVEGUIDE PROBLEMS

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**ABSTRACT.** We present a novel Windowed Green function (WGF) approach for the solution of the problem of wave propagation and radiation in open two- and three-dimensional waveguide structures containing waveguide junctions, inputs and terminations. The method is applicable to a variety of waveguide problems, including modeling of nonuniform waveguides with arbitrary junctions, illumination of semi-infinite waveguides, and radiation by dielectric antennas. The overall structures are illuminated by means of guided wave modes which are obtained by means of an efficient Maxwell eigensolver. The proposed WGF method, which is related to the approach proposed in [1] for scattering problems in presence of layered media without recourse to Sommerfeld integrals, is based on the use of an integral representation of the electromagnetic fields with densities defined over infinite boundaries, which are discretized only near the regions where the dielectric structure is nonuniform. A naive implementation based on direct truncation of the integrals mentioned above only yields low order convergence in terms of the size of the discretized region. The smooth window function restores super-algebraic convergence for the truncated integral operators, leading to a scheme that effectively treats the infinite structure as a finite scatterer without compromising accuracy near nonuniformities.

The near fields obtained by the WGF method can be used to compute the far fields on the basis of modal expansions—which generally must include both the discrete and continuous spectra. We address the most common challenges encountered when considering the continuous spectrum, which include the slowly (but oscillatory) decaying integrands, and singularities encountered at the transition between propagating and evanescent modes. The efficient, high order scheme we propose for evaluation of discrete and continuous Maxwell eigenmodes for open (dielectric) waveguides of arbitrary cross-section will also be presented as part of this talk. The discrete part of the spectrum is obtained by formulating the problem (a system of coupled Helmholtz equations for the  $z$ -components of the electric and magnetic fields) in terms of boundary layer potentials, and including an eigensearch algorithm which, following up on ideas proposed in [2], utilizes a certain “interior point” strategy to eliminate spurious modes and thus enable an efficient search for eigenvalues. For the continuous part of the spectrum, in turn, each mode is represented as a known incident field, plus an outgoing field [3], which is computed by solving the scattering problem that correctly enforces the electromagnetic boundary conditions.

**Keywords:** Windowed Green Function, Helmholtz Equation, Maxwell Eigenfunctions, Open Waveguides, Dielectric Antennas.

**Mathematics Subject Classifications (2010):** 78M15, 65R20

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