# SUPERCONVERGENT HDG METHODS FOR LINEAR THIRD-ORDER EQUATIONS IN ONE-SPACE DIMENSION

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ABSTRACT. We design and analyze *the first* hybridizable discontinuous Galerkin methods for stationary, third-order linear equations in one-space dimension. The methods are defined as discrete versions of characterizations of the exact solution in terms of local problems and transmission conditions. They provide approximations to the exact solution $u$ and its derivatives $q := u'$ and $p := u''$ which are piecewise-polynomials of degree $k_u$, $k_q$ and $k_p$, respectively. We consider the methods for which the difference between these polynomial degrees is at most two. We prove that all these methods have superconvergence properties which allows us to prove that their numerical traces converge at the nodes of the partition with order at least $2\,k + 1$, where $k$ is the minimum of $k_u, k_q, k_p$. This allows us to use an element-by-element post-processing to obtain new approximations for $u, q$ and $p$ converging with order at least $2k + 1$ uniformly. Numerical results validating our error estimates are displayed.

**Keywords**: HDG methods, superconvergent, third-order equations

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