DISCRETE HUYGENS POTENTIALS, A.K.A. TIME-DOMAIN ACOUSTICS WITH BEM

FRANCISCO-JAVIER SAYAS

Abstract. A very simple formula will start this talk:

$$(\mathcal{S} \star \lambda)(\mathbf{x}, t) := \int_\Gamma \frac{\lambda(\mathbf{y}, t - |\mathbf{x} - \mathbf{y}|)}{4\pi|\mathbf{x} - \mathbf{y}|} \mathrm{d}\Gamma(\mathbf{y}).$$

Here Γ is a any of the following geometric objects: the boundary of a closed bounded Lipschitz domain in the space, an open orientable Lipschitz surface, the finite disjoint union of several of the above. The function $\lambda : \Gamma \times \mathbb{R} \to \mathbb{R}$ acts as a density and it is assumed to be causal, that is, it vanishes for negative values of the time variable. This formula defines the single layer retarded acoustic potential, which is just the mathematical formulation of Huygens's description of waves traveling in space, moving on spherical surfaces, conserving energy and with no memory. I will discuss the past and present of mathematical and numerical analysis of this entity (and some other related potentials and integral operators), comparing what can be done in the Laplace domain and directly in the time domain.

The final aim is the full discretization of an equation of the form $\mathcal{S} \star \lambda = g$, for causal g, using Galerkin semidiscretization in space and automatic quadrature in time. This will lead to discussing several related topics: the Laplace domain theory of acoustic potentials (triggered by the pioneering work of Alain Bamberger and Tuong Ha-Duong), some basic aspects of the theory of operator-valued distributions, the many attractive features of Christian Lubich's Convolution Quadrature algorithm, and some powerful (while very elementary) tools that we can take from the theory of C_0-(semi)groups of operators applied to somewhat exotic transmission problems. The talk will include some hints at my contributions to the topic, including past and current collaboration with Antonio Laliena, Víctor Domínguez, Lehel Banjai, Christian Lubich and Sijiang Lu.

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Department of Mathematical Sciences, University of Delaware
E-mail address: fjsayas@math.udel.edu