SPARSE TENSOR EDGE ELEMENTS

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ABSTRACT. The second moment problem for linear operator equations $\mathsf{A}u = f$, $\mathsf{A} : H \to H'$, H a Hilbert space, with stochastic right hand side f leads to the operator equation $(\mathsf{A} \otimes \mathsf{A})\mathcal{M}^2 u = \mathcal{M}^2 f$ that features the tensor product operator $\mathsf{A} \otimes \mathsf{A}$ [4, Ch. 1]. If $V_h \subset V$ supplies a stable Galerkin discretization of A, then the *full tensor trial space* $V_h \otimes V_h$ will spawn a stable Galerkin method for $\mathsf{A} \otimes \mathsf{A}$. Unfortunately, $\dim(V_h \otimes V_h) = (\dim V_h)^2$, whereas the approximation power of $\dim V_h^{(2)}$ is usually not better than that of V_h. This is the notorious "curse of dimensionality". Taking for granted smoothness of $\mathcal{M}^2 u$, a remedy is offered by sparse tensor Galerkin discretization, using subspaces $\widehat{V}_h^{(2)}$ of $V_h^{(2)}$ with approximation power almost like that of V_h, but dimensions substantially reduced to $\dim \widehat{V}_h^{(2)} \approx \dim V_h$, see [4, Section 1.4]. However, the stability of sparse tensor Galerkin discretizations can no longer be inferred from that for V_h applied to A, unless A is positive. Non-positive operators are invariably encountered in wave propagation phenomena in frequency domain, and for them stability of the sparse tensor Galerkin discretization has to be established directly. This was done for boundary value problems for the Helmholtz equation $-\Delta u - k^2 u = f$ in [5], see also [4, Sect. 1.4]. The proposed presentation will discuss uniform asymptotic stability of tensor product Maxwell cavity operators discretized by means of sparse tensor edge elements. Besides being non-positive, these operators enjoy only a generalized Gårding inequality, which entails using a sparse tensor version of the edge element Fortin projectors [1]. Then, provided that the sparse tensor spaces comply with a uni-directional minimal resolution condition, we can show uniform asymptotic quasi-optimality of sparse tensor Galerkin solutions. The techniques can also be adapted to the electric field integral equation (EFIE) [3].

Keywords: Sparse tensor approximation, stochastic source problems, Maxwell cavity source problem, edge elements, Fortin projector, commuting diagram property

Mathematics Subject Classifications (2000): 78M15, 65C30, 65N38, 60H35

References

[1] D. BOFFI, *Fortin operator and discrete compactness for edge elements*, Numer. Math., 87 (2000), pp. 229–246.

[2] R. HIPTMAIR, C. JEREZ-HANCKES, AND C. SCHWAB, *Sparse tensor edge elements*, Report 2012-34, SAM, ETH Zürich, Switzerland, 2012.

[3] R. HIPTMAIR AND C. SCHWAB, *Natural boundary element methods for the electric field integral equation on polyhedra*, SIAM J. Numer. Anal., 40 (2002), pp. 66–86.

[4] C. SCHWAB AND C. J. GITTELSON, *Sparse tensor discretizations of high-dimensional parametric and stochastic PDEs*, Acta Numerica, 20 (2011), pp. 291–467.

[5] T. VON PETERSDORFF AND C. SCHWAB, *Sparse finite element methods for operator equations with stochastic data*, Appl. Math., 51 (2006), pp. 145–180.

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