LAGRANGIAN AND SEMI-LAGRANGIAN GALERKIN METHODS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. We present a unified approach to state and analyze Pure Lagrangian and Semi-Lagrangian methods for solving convection-diffusion partial differential equations. When combined with finite element methods for space discretization, the Semi-Lagrangian schemes are also called Lagrange-Galerkin or characteristics-finite element methods. They have been introduced in the eighties by Pironneau[4] and Douglas-Russel[3]. Our approach uses the formalism of continuum mechanics in which classical and new methods can be introduced in a natural way. For convection-diffusion equations, second order Pure Lagrange-Galerkin schemes have been introduced by the authors in [1] and [2] where stability and error estimates for time semi-discretized and full-discretized schemes have been proved. When applied to the Navier-Stokes equations, the above ideas lead to displacement methods similar to those used for numerical solution of solid mechanics problems. Unfortunately, the Pure Lagrangian methods seem not to work well for real fluid mechanics problems but the Semi-Lagrangian schemes yield good results. In particular, these new displacement methods can be useful for solving fluid-structure interaction problems (e.g. in aero-elasticity) because fluid-solid coupling at the interphase is straightforward. While the numerical analysis appears to be difficult, preliminary numerical results for test examples exhibit good properties.

Keywords: Lagrangian methods. Semi-Lagrangian methods. Lagrange-Galerkin methods. Characteristics methods. Convection-diffusion equations. Navier-Stokes equations. Stability. Error estimates.

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