

# A PDE APPROACH TO FRACTIONAL DIFFUSION: A PRIORI AND A POSTERIORI ERROR ANALYSES

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ABSTRACT. The purpose of this work is the study of solution techniques for problems involving fractional powers of [the Dirichlet Laplace] symmetric coercive elliptic operators in a bounded domain with Dirichlet boundary conditions. This operator can be realized as the Dirichlet to Neumann map of a degenerate/singular elliptic problem posed on a semi-infinite cylinder, which we analyze in the framework of weighted Sobolev spaces. Motivated by the rapid decay of the solution of this problem, we propose a truncation that is suitable for numerical approximation. We discretize this truncation using first degree tensor product finite elements. We derive suboptimal a priori error estimates for shape regular discretizations and optimal error estimates for anisotropic discretizations, both estimates in weighted Sobolev spaces. Next, as a first step to design an adaptive algorithm, we present a computable a posteriori error estimator, which relies on the solution of small discrete problems on stars. It exhibits built-in flux equilibration and is equivalent to the energy error up to data oscillation. A simple adaptive strategy is designed, which reduces error and data oscillation. We present numerical experiments to illustrate the a priori and a posteriori error estimates as well as the adaptive method's performance.

**Keywords:** Fractional diffusion; finite elements; nonlocal operators; degenerate and singular equations; anisotropic elements.

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## REFERENCES

- [1] L. Caffarelli and L. Silvestre. An extension problem related to the fractional Laplacian. *Comm. Partial Differential Equations*, 32: 1245-1260, 2007.
- [2] A. Capella, J. Dávila, L. Dupaigne and Y. Sire. Regularity of radial extremal solutions for some non-local semilinear equations. *Comm. Partial Differential Equations*, 36: 1353–1384, 2011.
- [3] R. Durán and A. Lombardi. Error estimates on anisotropic  $Q_1$  elements for functions in weighted Sobolev spaces. *Math. Comp.*, 74:1679-1706, 2005.
- [4] P. Morin, R.H. Nochetto and K. Siebert. Local problems on stars: a posteriori error estimators, convergence, and performance. *Math. Comp.*, 72:11067–1097, 2003.
- [5] P.R. Stinga and J.L Torrea. Extension problem and Harnack's inequality for some fractional operators. *Comm. Partial Differential Equations*, 35:2092-2122, 2010.

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