AN EXISTENCE RESULT FOR AN ELECTROMAGNETIC LEVITATION MODEL IN CYLINDRICAL COORDINATES

RAFAEL MUÑOZ-SOLA, CARLOS REALES, AND RODOLFO RODRÍGUEZ

Abstract. The aim of this talk is to present an existence result for a model of the levitation of a cylindrical metallic body subjected to an electromagnetic field generated by a cylindrical coil. We consider the transient eddy current equations in cylindrical coordinates (r, θ, z). We assume the density current in the coil $\vec{J}_S := J_S(t, r, z)\,\vec{e}_\theta$ is a data and all the physical quantities are independent of θ. We introduce a magnetic vector potential $\vec{A} := A(t, r, z)\,\vec{e}_\theta$ and proceed as in [1] to reduce this model to the following degenerate parabolic PDE:

$$\sigma \frac{\partial A}{\partial t} \vec{e}_\theta + \nabla \times \left[\frac{1}{\mu} \nabla \times (A\vec{e}_\theta) \right] = J_S \vec{e}_\theta \qquad \text{in } (0,T) \times \Omega,$$

where $\Omega := (0, r_{max}) \times (z_{min}, z_{max})$ lies in the meridian half-plane (r, z), $\mu > 0$ is a constant, $\Omega_t := \Omega_0 + u(t)\,\vec{e_z}$ is the location of the metallic piece at time t, σ is the electric conductivity of the metallic piece in Ω_t and $\sigma = 0$ outside Ω_t. Moreover, we assume that $\sigma(t, r, z) := \widehat{\sigma}((r, z) - u(t)\,\vec{e_z})$, where $\widehat{\sigma} \in L^\infty(\Omega_0)$ and $\widehat{\sigma} \geq \underline{\sigma} > 0$. We complement the PDE with the boundary condition $A = 0$ in $(0, T) \times \partial\Omega$ and the initial condition $A|_{\{0\}\times\Omega_0} = A^0$.

The vertical displacement u of the metallic piece is governed by the initial value problem

$$m \frac{d^2u}{dt^2}(t) = 2\pi \int_{\Omega_t} f_z(t, r, z)\, r\, dr\, dz, \qquad \frac{du}{dt}(0) = v_0, \qquad u(0) = 0,$$

where f_z is the vertical component of the Lorentz force, which in this case is given by

$$f_z = -\sigma \, \frac{\partial A}{\partial t} \, \frac{\partial A}{\partial z} \quad \text{in } \Omega_t.$$

Under appropriate assumptions, we have shown in [2] that the PDE above has a weak solution $A = A(u)$ for each given $u \in C^1([0, T])$. Under the additional assumption that the set of discontinuities of $\widehat{\sigma}$ has measure zero, we prove that the coupled problem has a solution locally in time. With this end, we apply Schauder's theorem to the mapping $F : u \in C^1([0, T]) \mapsto w$, where $w \in C^1([0, T])$ denotes the solution of the initial value problem above with $A = A(u)$.

Keywords: levitation, transient eddy current problems, axisymmetric problems, degenerate parabolic problems.

Mathematics Subject Classifications (2000): 35K65.

References

[1] A. Bermúdez, C. Reales, R. Rodríguez, and P. Salgado Numerical analysis of a transient eddy current axisymmetric problem involving velocity terms. *Numer. Methods PDEs*, 28 (2012) 984–1012.

[2] A. Bermúdez, R. Muñoz-Sola, C. Reales, R. Rodríguez, and P. Salgado. *Mathematical and numerical analysis of a transient eddy current problem arising from electromagnetic forming.* Preprint DIM 2011-07, Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, 2011.

Departamento de Matemática Aplicada. Universidade de Santiago de Compostela, 15706, Santiago de Compostela, Spain
E-mail address: rafael.munoz@usc.es

Departamento de Matemáticas y Estadística, Universidad de Córdoba, Montería, Colombia
E-mail address: creales@correo.unicordoba.edu.co

$\mathrm{CI^2MA}$, Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile
E-mail address: rodolfo@ing-mat.udec.cl