SIMPLE COMPUTATION OF REACTION-DIFFUSION PROCESSES ON POINT CLOUDS

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Abstract. Partial Differential Equations (PDEs) are widely used to describe continuum processes such as diffusion, chemical reactions, fluid flow or electrodynamics. In standard 3D settings, these take a familiar PDE form, such as a reaction-diffusion equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial\, x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + f(u)$$

and the ways to numerically solve such equations are well-developed. The study of reaction-diffusion processes is more complicated on general curved surfaces than on standard Cartesian coordinate spaces. Moreover, in practical settings, surfaces are often defined simply as a set of points — a point cloud — sampled from the underlying surface. Because the connectivity of the points is not provided, this adds further complexity to methods that need to reconstruct the geometric properties of the surface, such as the metric distance. Here we show how to solve reaction-diffusion equations directly on a point-cloud that represents the underlying surface in a way that reduces the problem to working with the entirely standard classical 3D discretizations and solver libraries. The method is independent of the codimension of the surface, and does not reconstruct the point cloud into an organized triangulation or surface representation.

Keywords: Reaction-diffusion process, surface diffusion, Laplace–Beltrami operator, surface computation, embedding methods, Closest Point Method.

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