DERIVATION AND NUMERICAL STUDY OF RELATIVISTIC BURGERS EQUATIONS ON A CURVED SPACETIME

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ABSTRACT. We consider nonlinear hyperbolic balance laws posed on a curved spacetime endowed with a volume form and identify a unique (up to normalization) hyperbolic balance law that enjoys the Lorentz invariance property also shared by the Euler equations of relativistic compressible fluids. This model is unique up to normalization and converges to the standard inviscid Burgers equation in the limit of infinite light speed. Moreover, from the Euler system of relativistic compressible flows on a curved background, we derive both the standard inviscid Burgers equation and our relativistic generalizations. The proposed models are referred to as relativistic Burgers equations on curved spacetimes. We then introduce a finite volume scheme for the approximation of discontinuous solutions to these relativistic Burgers equations. Our scheme is formulated geometrically and is consistent with the natural divergence form of the balance law and applies to weak solutions containing shock waves. Most importantly, our scheme is well-balanced in the sense that it preserves static equilibrium solutions. Numerical experiments demonstrate the convergence of the proposed finite volume scheme and its relevance for computing late-time asymptotics of (possibly) discontinuous solutions on a curved background.

This presentation is based on the joint paper [2]

Keywords: tnonlinear hyperbolic, balance law, curved spacetime, relativistic Burgers equation, well balanced

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