BOUNDARY CONTROL OF BIDOMAIN EQUATIONS IN CARDIAC ELECTROPHYSIOLOGY

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Abstract. The focus of this work is on the development and implementation of an efficient numerical techniques to solve an optimal control problem related to a reaction-diffusion system arising in cardiac electrophysiology. The bidomain equations are widely accepted as one of the most complete descriptions of the cardiac bioelectric activity at the tissue and organ level. The model consist of a system of elliptic partial differential equations coupled with a non-linear parabolic equation of reaction-diffusion type, where the reaction term, modeling ionic transport is described by a set of ordinary differential equations. In particularly, we use the Fenton-Karma ionic model in our simulation study. In this ionic model, the source functions are discontinuous in state space solution.

The optimal control problem is considered as a PDE constrained optimization problem. The optimal control approach is based on minimizing a properly chosen cost functional $J(v, I_e)$ depending on the extracellular current I_e as input, which must be determined in such a way that wavefronts of transmembrane voltage v are smoothed in an optimal manner, where the transmembrane potential v as one of the state variables, see [1, 2]. The optimal control formulation is presented and a formal derivation of the adjoint equations and the first order optimality conditions, which are the basis for numerical solution are provided. The derivative of discontinuous source functions, due to the Fenton-Karma model, is required to solve the adjoint equations, which poses a lot of computational challenges. The efficient numerical techniques for boundary control of the bidomain model as well as computational techniques to track the derivatives of discontinuous source functions in adjoint equations will be demonstrated. The numerical realization is described in detail and numerical experiments, which demonstrate the capability of influencing and terminating reentry phenomena, are presented. We employ the parallelization techniques to enhance the solution process of optimality system and a numerical feasibility study in a parallel environment will be shown.

Keywords: bidomain model, Fenton-Karma ionic model, defibrillation, optimal control, derivatives of Heaviside functions, parallel FEM, NCG algorithm.

Mathematics Subject Classifications (2000): 35K57, 49J20, 65Y05, 80M10

References

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