ADAPTIVE MULTIRESOLUTION DISCONTINUOUS GALERKIN SCHEMES FOR CONSERVATION LAWS

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Abstract. In the present work, we develop the concept of multiresolution-based DG (MR-DG) schemes [4], where for the sake of analysis we focus on scalar one-dimensional conservation laws. We emphasize that none of the conceptual ingredients are restricted to this setting but can be extended straight-forwardly to multidimensional systems of conservation laws. The key idea is to apply data compression by means of a multiresolution analysis (MRA) based on Alpert's multiwavelets [1, 2] and hard thresholding. Instead of performing time evolution on the full set of equations on the uniform reference grid, we only evolve the equations of a reduced set of significant multi-scale coefficients corresponding to a locally refined grid. Then we may introduce the *perturbation error* as the difference of the results obtained by performing computations with the reference DG scheme on a fully refined grid and the adaptive scheme. From a mathematical point of view it is of interest whether the perturbation error can be estimated by the threshold value. For scalar conservation laws we verify an a priori error estimate of the perturbation error. Since error estimates for the discretization error of the reference DG scheme are available in this case, cf. [3], the threshold value can be chosen such that the perturbation error and the discretization error are balanced. Due to the lack of similar error estimates for general systems of conservation laws, this choice may motivate the selection of a reasonable threshold value for practical problems. Recently, this was investigated numerically in [5] by performing parameter studies in case of the Euler equations.

Keywords: Conservation laws, Discontinuous Galerkin schemes, grid adaptation, multi-wavelets.

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References

[1] B. Alpert. A class of bases in L^2 for the sparse representation of integral operators. *SIAM J. Math. Anal.*, 24:246–262, 1993.

[2] B. Alpert, G. Beylkin, D. Gines and L. Vozovoi. Adaptive solution of partial differential equation in multi-wavelet bases. *J. Comp. Phys.*, 182:149–190, 2002.

[3] B. Cockburn and C.-W. Shu. TVB Runge-Kutta local projection discontinuous Galerkin finite element method for conservation laws II: General framework. *Mathematics of Computation*, 52(186):411–435, 1989.

[4] N. Hovhannisyan, S. Müller and R. Schäfer. Adaptive multiresolution Discontinuous Galerkin schemes for conservation laws. *Mathematics of Computation*, accepted, 2012.

[5] F. Iacono, G. May, S. Müller and R. Schäfer. A high-order discontinuous Galerkin discretization with multiwavelet-based grid adaptation for compressible flows. AICES Preprint, AICES-2011/08-1, 2011.

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