AN IMPROVED TIME DOMAIN LINEAR SAMPLING METHOD FOR ROBIN AND NEUMANN OBSTACLES

HOUSSEM HADDAR, ARMIN LECHLEITER, AND SIMON MARMORAT

Abstract. We present and study a time domain linear sampling method as an algorithm to solve the inverse scattering problem of reconstructing an obstacle with Robin or Neumann boundary condition from time-dependent near-field measurements of scattered waves. Our algorithm is an improved version of the one introduced in [1] to solve a similar inverse scattering problem for obstacles with Dirichlet boundary conditions. This algorithm is a direct imaging technique able to provide geometric information on the obstacle from indicated measurements in a rather direct way. The main components of the method are the near-field operator, a linear integral operator that takes the measured data as integral (and convolution) kernel, and special test functions that are constructed via singular solutions to the wave equation. Using these ingredients, the method examines whether a point belongs to the obstacle by checking whether these test functions belong to the range of the measurement operator. Plotting the reciprocal of the norm of the corresponding pre-image yields a picture of the scattering object.

Apart from the analysis of a different scattering problem, we propose in the present work an improvement of the method originally introduced in [1] on both theoretical and numerical levels. More precisely, we shall analyse the method for incident waves generated by pulses that have bounded frequency spectrum. Furthermore, adapting the function space setting to this type of data allows us to provide a simpler analysis. On the numerical side, we shall present a fast implementation of the inversion algorithm that relies on a FFT-based evaluation of the near-field operator. We also show, by mixing the use of monopoles and dipoles as test functions, the possibility of simultaneously reconstructing Dirichlet, Neumann and Robin obstacles.

Keywords: linear sampling method, time domain, boundary condition

Mathematics Subject Classifications (2000): 65N21, 65R32, 78A46

References

[1] Q. Chen, H. Haddar, A. Lechleiter and P. Monk.
A sampling method for inverse scattering in the time domain.
Inverse Problems, 26:085001, 2010

[2] M. Costabel.
Time-dependent problems with the boundary integral equation method.
In: *Encyclopedia of Computational Mechanics*. Proceedings ENUMATH 2004.

[3] T. Ha-Duong.
On retarded potential boundary integral equations and their discretisations.
In *Topics in Computational Wave Propagation: Direct and Inverse Problems*.
M. Ainsworth, P. Davies, D. Duncan, P. Martin and B. Rynne (eds), pp. 301-336, Springer-Verlag.

[4] A. Bamberger and T. Ha Duong.
Formulation variationnelle espace-temps pour le calcul par potentiel retardé de la diffraction d'une onde acoustique.
Mathematical Methods in the Applied Science, vol. 8, pp. 405-435, 1986.

DEFI, INRIA Saclay–Ile-de-France and Ecole Polytechnique, 91128 Palaiseau, France
E-mail address: haddar@cmap.polytechnique.fr

Center for Industrial Mathematics, University of Bremen, 28359 Bremen, Germany
E-mail address: lechleiter@math.uni-bremen.de

CEA LIST, Centre de Saclay - point courrier 120, 91191 Gif-sur-Yvette, France
E-mail address: simon.marmorat@inria.fr