ANALYSIS OF AN ASYMPTOTIC PRESERVING SCHEME FOR RELAXATION SYSTEMS

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ABSTRACT. We present the work of [3]. We consider an asymptotic preserving numerical scheme initially proposed by F. Filbet & S. Jin [2] and G. Dimarco & L. Pareschi [1] in the context of nonlinear and stiff kinetic equations. Here, we propose a convergence analysis of such a scheme for the approximation of a system of transport equations with a nonlinear source term, for which the asymptotic limit is given by a conservation laws. Precisely, the general form of the investigated system is:

$$\left\{\begin{array}{l}\partial_t u^\varepsilon + \partial_x v^\varepsilon = 0\,, \quad u^\varepsilon(0,x) = u_0^\varepsilon(x)\\ \\ \partial_t v^\varepsilon + a\,\partial_x u^\varepsilon = -\dfrac{1}{\varepsilon}\,\mathcal{R}(u^\varepsilon, v^\varepsilon), \quad v^\varepsilon(0,x) = v_0^\varepsilon(x),\end{array}\right. \tag{1}$$

where $a > 0$ is a constant coefficient to be discussed later, ε is the relaxation parameter, which can be either large or small (leading to a stiff source term) and $\mathcal{R} : \mathbb{R}\times\mathbb{R} \mapsto \mathbb{R}$ is a nonlinear function such that $\mathcal{R}(0,0) = 0$. This system of equations is often referred to as a two velocity kinetic model.
We investigate the convergence of the approximate solution $(u_h^\varepsilon, v_h^\varepsilon)$ to (1), where h represents the discretization parameter. Uniform convergence with respect to ε and h is proved and error estimates are also obtained. Finally we apply our scheme to the Broadwell model in order to illustrate the efficiency of the method.

Keywords: asymptotic preserving scheme, finite volume, stiff source, convergence of numerical scheme, hyperbolic relaxation system.

Mathematics Subject Classifications (2000): 35L45, 65M12, 82C40.

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