SPLITTING METHOD FOR HYPERBOLIC SYSTEMS WITH STIFF RELAXATION SOURCE TERMS: ANALYSIS AND APPLICATION TO POLYDISPERSE SPRAYS IN SOLID ROCKET MOTORS

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Abstract. Many physical problems encountered in multiphase flows are described by hyperbolic systems of conservation laws with relaxation terms. These systems combine both nonlinear interaction and convection. In the context of dispersed two-phase flows, the Eulerian Multi-Fluid model [1, 2] is particularly suitable to describe polydisperse sprays in turbulent gaseous flows. In solid rocket motor applications, high particle mass loadings can appreciably modify the gaseous flow field due to momentum and energy exchanges. This two-way coupling between both phases is expressed through relaxation source terms. The range of particle sizes can be quite wide and the relaxation times i.e. the characteristic times for particles to reach a thermal or dynamic equilibrium depends on their size parameter. For small particles with fast time scales, the relaxation terms become stiff. Diffusive equations, considering that the asymptotic regime is reached, could be envisionned for some subrange of particles/droplets of very small inertia; however, taking into account the whole size range of a polydisperse spray is incompatible with such an approach.

Splitting techniques consisting in decoupling the convection and relaxation terms have several advantages such as the use of dedicated numerical schemes. But they can decrease the order of accuracy when there are small time scales [3, 4]. A novel numerical strategy based on a time cost saving operator splitting has been developed and implemented in CEDRE, an industrial-oriented research code developed at ONERA. It demonstrates very promising results in terms of physical response for solid rocket motor applications [5]. We show that this splitting method is accurate for non stiff source terms but introduces a certain level of error when the time step is larger than particle relaxation times, even if high order adaptive schemes are used in order to integrate the stiff source terms. Such technique requires very small time step to capture asymptotic regimes with full accuracy.

The purpose of the present contribution is first to isolate the error of the splitting scheme from those introduced by the convection and relaxation schemes. We provide a characterization of the splitting error for simplified acoustic/spray interactions. Then we estimate the feasibility of the splitting method by evaluating the impact of such an error in the context of realistic solid propulsion simulations. Moreover we use our analysis of the splitting error to extend the splitting technique and to predict accurately the asymptotic regime. Numerical tests are proposed to compare the classical splitting with the new strategy introduced.

References

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