GENERALIZED PLANE WAVE METHODS FOR MAXWELL'S EQUATION MODELLING REFLECTOMETRY IN FUSION PLASMAS.

BRUNO DESPRÉS AND LISE-MARIE IMBERT-GÉRARD

Abstract. We will present our last results on the theoretical and computational solutions of a certain Maxwell's equation that arises in physics of plasmas.

Reflectometry is an electromagnetic probing process for fusion plasmas, see [1]. For ITER project [5], numerical simulations are required before performing any experiment. Since the volume of ITER reactor is expected to be 840 cubic meters, physicists need efficient numerical methods to be developed instead of the current finite different schemes they are currently using. Reflectometry is modelled by Maxwell's equation with a specific dielectric tensor. This equation split into two different propagation modes, namely ordinary (O) and extraordinary (X). The O-mode equation is Helmholtz equation with a smooth vanishing coefficient. The X-mode equation is much more tricky, and known in the physics community to have a singular solution. We theoretically studied this equation in [3] We were able to define a singular solution, obtained via a limit absorption principle.

The numerical methods we developed in [4] for reflectometry, based on an ultra-weak variational formulation [2], are adapted to the main unusual feature of these equations : the coefficients vanish smoothly at some points, called a cut-off. This makes any standard plane wave method irrelevant since the local wave number vanishes at the cut-off. We designed generalized plane wave basis functions to overcome this problem and achieve high order convergence, the order being linked to order of approximation of the coefficient. They are in fact exponential of polynomials, and the polynomials coefficients are fitted to ensure that the function approximates a solution of the homogeneous equation, which is required by the ultra-weak variational formulation.

We will explain precisely the design process for these basis functions and the main interest of such basis functions, including approximation properties.

Keywords: High order numerical methods, generalized plane waves.

References

[1] F. Da Silva, S. Heuraux, M. Manso, Developments on reflectometry simulations for fusion plasmas: application to ITER position reflectometers In: *J. Plasma Physics*, vol. 72, part 6, pp. 12051208, 2006.

[2] B. Després, Sur une formulation variationnelle de type ultra-faible. (French) [An ultra-weak-type variational formulation]. C. R. Acad. Sci. Paris Sr. I Math. 318 (1994), no. 10, 939944.

[3] B. Després, L.-M. Imbert-Gérard, R. Weder, Hybrid resonance of Maxwells equations in slab geometry (submitted).

[4] L.-M. Imbert-Gérard, B. Després, A generalized plane wave numerical method for smooth non constant coefficients (submitted).

[5] http://www.iter.org/, ITER project.

Laboratoire Jacques-Louis Lions, UPMC - Paris 6, Boite courrier 187, 75252 Paris cedex 05
E-mail address: `despres@ann.jussieu.fr`

Laboratoire Jacques-Louis Lions, UPMC - Paris 6, Boite courrier 187, 75252 Paris cedex 05
E-mail address: `imbert@ann.jussieu.fr`