A SPECTRAL ANALYSIS OF THE COMBINATION OF OSRC PRECONDITIONER AND FAST MULTIPOLE METHOD FOR 3-D HIGH-FREQUENCY SCATTERING

MARION DARBAS, ERIC DARRIGRAND, AND YVON LAFRANCHE

Abstract. Integral equation methods are widely used to study wave propagation outside a bounded 3-D domain. Thanks to these techniques, the governing boundary-value problem is reduced to an integral equation on the surface Γ of the scatterer ([3]). For acoustic scattering, under some assumptions on the regularity of Γ and the incident wave, the Combined Field Integral Equation (CFIE) is uniquely solvable in $H^{1/2}(\Gamma)$ for all frequency $k > 0$, and involves the first and second traces of the double-layer potential respectively denoted M and D. We apply a preconditioning strategy to the strongly singular and non-compact operator D by introducing an approximation $\widetilde{V}$ of the exterior Neumann-to-Dirichlet map, using On-Surface Radiation Condition (OSRC) methods ([1]). The new equation is uniquely solvable for all wavenumbers and exhibits very interesting spectral properties. The OSRC technique only involves local operators, so that the numerical implementation of the preconditioning operator requires only the use of a sparse direct solver and does not really affect the cost of the iterative resolution of the CFIE equation. The most expensive part of the resolution still comes from the integral operators. The Fast Multipole Method (FMM) ([2]) is then considered to deal with the operators M and D. For the acoustic case, a thorough study of the eigenvalues behavior illustrates the impact of the OSRC-preconditioning technique on the spectrum of the CFIE operator for different geometries: sphere, cube, cube with rectangular cavity, cone-sphere, trapping c-shape domain, submarine). All the tests confirm a significant reduction of the condition number, and the convergence of the solver (GMRES) corroborates the spectral analysis. Only a few GMRES iterations are required for both high frequencies and refined meshes. For the electromagnetic case, under study, we expect the same eigenvalues behavior. A theoretical study were done in [4] and a first promising application is considered in [5].

Keywords: integral equations, OSRC, FMM, high-frequency scattering, Helmholtz, Maxwell

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LAMFA – Université de Picardie
E-mail address: marion.darbas@u-picardie.fr

IRMAR – Université de Rennes 1
E-mail address: eric.darrigrand-lacarrieu@univ-rennes1.fr

IRMAR – Université de Rennes 1
E-mail address: yvon.lafranche@univ-rennes1.fr