PARALLEL MULTILEVEL SOLVERS FOR THE CARDIAC ELECTROMECHANICAL COUPLING

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Abstract. We construct a parallel solver for the cardiac electromechanical coupling. The electric model consists of two non-linear parabolic partial differential equations (PDEs), the so-called Bidomain model [1, 2], which describes the spread of the electric impulse in the heart muscle. The two PDEs are coupled through the reaction term with the membrane model, a stiff system of ordinary differential equations, modeling the cellular membrane ionic currents. The electric model is coupled with a non-linear elastic model, where the myocardium is considered as a nearly-incompressible transversely isotropic hyperelastic material. The discretization of the whole electromechanical model is performed by Q1 finite elements in space and a semi-implicit finite difference scheme in time. This approximation strategy yields at each time step the solution of a large scale ill-conditioned linear system deriving from the discretization of the Bidomain model and a non-linear system deriving from the discretization of the finite elasticity model. The parallel solver developed consists of solving the linear system with the Conjugate Gradient method, preconditioned by a Multilevel Additive Schwarz preconditioner, and the non-linear system with the Newton method, where the Jacobian system is solved by GMRES, preconditioned by an Algebraic Multigrid preconditioner. Two- and three-dimensional parallel numerical tests on a Linux cluster show that the parallel solver proposed is scalable and robust with respect to the domain deformations induced by the cardiac contraction.

Keywords: cardiac electromechanical coupling; Bidomain model; Multilevel Additive Schwarz preconditioner.

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