SEMI-IMPLICIT NUMERICAL MODELLING OF AXIALLY SYMMETRIC FLOWS IN COMPLIANT ARTERIAL SYSTEMS

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Abstract. This talk is concerned with the design and application of very efficient numerical methods for the simulation of blood flow in complex networks of medium to large arteries in mammals in general and, in particular, in humans. The task at hand involves the solution of two coupled problems, namely (a) the interior blood flow distribution within the arterial system and (b) the determination of the domain itself given by the geometrical configuration of the compliant arteries [2]. For medium-to-large arteries the fluid dynamical problem (a) is accurately modeled by the three-dimensional, time-dependent incompressible Navier-Stokes equations. Problem (b) concerns the compliant behavior of the arterial walls and requires a model for their viscoelastic properties. The complete coupled problem is rather complex and in practice simpler models are resorted to. Even one-dimensional models are still quite useful, especially if these are used to describe a complex network of vessels in the so-called multiscale framework [3], where more complex models are used at junctions and other locations where the one-dimensional assumption is thought to be inadequate. In classical 1D models the coupling of problems (a) and (b) is achieved via an integrated form of the continuity equation across the radial direction coupled to an assumed tube law describing a simple relation between pressure and vessel radius. Here we start from the incompressible time-dependent Navier-Stokes equations in three space dimensions and assume axial symmetry. As the axial scale is typically much larger than the radial scale, a dimensional analysis shows that the pressure can be assumed to be in (radial) hydrostatic equilibrium; see Ref. [4], for example. Regarding our new numerical method proposed in [1] we adopt the semi-implicit approach proposed by Casulli [5, 6, 7]. We note in passing that the choice of the time step in an explicit numerical method for solving the problem would be restricted by the Courant-Friedrichs-Lewy (CFL) stability condition that relates the time step size to the spatial mesh size and the maximum wave speed. Further restrictions would result from the viscosity wall friction terms. On the other hand, a fully implicit method would be far too complex, mainly because of its nonlinearity. Within the above framework, the CFL condition on the maximum wave speed can be circumvented if a semi-implicit discretization is used to approximate the pressure term in the momentum equation and the velocity in the vessel-wall equation Furthermore, the semi-implicit discretization is also extended to the viscosity term and to the wall friction. An Eulerian-Lagrangian method is proposed for treating the nonlinear advective terms. Finally, the nonlinearity that arises from the time discretization of the vessel-wall equation is conveniently solved by an iterative Newton type algorithm. The resulting numerical scheme is exceedingly efficient and thus suitable for ambitious, large scale practical hemodynamical applications. At the end of the talk, an extension of the present method to *non–hydrostatic* and fully *three–dimensional* flows in curved compliant tubes will be presented.

Keywords: blood flow; compliant arteries; moving boundaries; axially symmetric flow; hydrostatic equilibrium; semi-implicit method; finite difference; finite volume

References

[1] V. Casulli and M. Dumbser and E.F. Toro. Semi-implicit numerical modeling of axially symmetric flows in compliant arterial systems. *International Journal for Numerical Methods in Biomedical Engineering*, 28:257–272, 2012.
[2] Y. C. Fung. *Biomechanics: Circulation.*. Springer New York, Berlin, Heidelberg, 2010
[3] A. Quarteroni and M. Tuveri and A. Veneziani. Computational vascular fluid dynamics: problems, models and methods. Survey article. *Comput. Visual Sci*, 2:163–197, 2000.

[4] A.C.L. Barnard and W.A. Hunt and W.P. Timlake and E. Varley. A theory of fluid flow in compliant tubes. *Biophysical Journal*, 6:717–724, 1966.

[5] V. Casulli. Semi-implicit finite difference methods for the two-dimensional shallow water equations. *J. Comput. Phys.*, 86:56–74, 1990.

[6] V. Casulli. A high-resolution wetting and drying algorithm for free-surface hydrodynamics. *Int. J. Numer. Meth. Fluids*, 60:391–408, 2009.

[7] V. Casulli and G.S. Stelling. Semi-implicit subgrid modelling of three-dimensional free-surface flows. *Int. J. Numer. Meth. Fluids*, 60:391–408, 2011.

[8] J.R. Womersley. Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known. *Journal of Physiology*, 127:553–563, 1955.

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