A HYBRID STOCHASTIC GALERKIN METHOD FOR A CLARIFIER-THICKENER MODEL WITH RANDOM FEED

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Abstract. Uncertainty quantification techniques are often required if the values of the parameters are not given exactly. The straightforward Monte-Carlo computations of sampling solutions are easily implemented, but computationally inefficient due to slow convergence. The polynomial chaos expansion yields a coupled deterministic system that allows faster computations. In particular from the point of view of the parallelisation this method is inefficient with increasing polynomial order.

We present a hybrid stochastic Galerkin (HSG) method for the expression of the uncertainty. The HSG method combines polynomial chaos and multi-wavelet representation. The HSG method yields a partially decoupled deterministic system, that allows efficient parallelisation. The complexity of the problem can be further reduced by stochastic adaptivity.

To validate the approach we consider the continuous sedimentation process in a clarifier-thickener described by a scalar nonlinear conservation law for the local solids volume fraction. We discuss the application of the adaptive HSG finite volume method on the clarifier-thickener problem with random feed and present the numerical results.

Keywords: clarifier-thickener model, polynomial chaos, stochastic Galerkin, uncertainty quantification, finite volume method

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References

[1] R. Abgrall. A simple, flexible and generic deterministic apporach to uncertainty quantifications in non linear problems: application to fluid flow problems. 2007.

[2] R. Bürger, I. Kröker, and C. Rohde. A hybrid stochastic galerkin method for uncertainty quantification applied to a conservation law modelling a clarifier-thickener unit. *ZAMM*. submitted.

[3] R. Bürger, R. Ruiz-Baier, K. Schneider, and H. Torres. A multiresolution method for the simulation of sedimentation in inclined channels. *Int. J. Numer. Anal. Model.*, 9:479–504, 2012.

[4] G. Poëtte, B. Després, and D. Lucor. Uncertainty quantification for systems of conservation laws. *J. Comput. Phys.*, 228(7):2443–2467, 2009.

[5] I. Kröker R. Bürger and C. Rohde. Uncertainty quantification for a clarifier-thickener model with random feed. In *Finite volumes for complex applications VI*, volume 1, pages 195–203. Springer, 2011.

[6] J. Tryoen, O. Le Maître, M. Ndjinga, and A. Ern. Intrusive Galerkin methods with upwinding for uncertain nonlinear hyperbolic systems. *J. Comput. Phys.*, 229(18):6485–6511, 2010.

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