# CONVEXITY-PRESERVING FLUX IDENTIFICATION FOR SCALAR CONSERVATION LAWS MODELLING SEDIMENTATION

RAIMUND BÜRGER AND STEFAN DIEHL

Abstract. The sedimentation of a suspension of small particles dispersed in a viscous fluid can be described by a scalar, nonlinear conservation law (the $x$-axis is pointing upwards)

$$\frac{\partial \phi}{\partial t} - \frac{\partial f_{\mathrm{b}}(\phi)}{\partial x} = 0,$$

whose flux function, the so-called batch-settling flux $f_{\mathrm{b}}$, usually has one inflection point. The identification of $f_{\mathrm{b}}$ is a problem of theoretical interest and practical importance for the implementation of numerical schemes for continuous sedimentation. Inverse problems where (a portion of) $f_{\mathrm{b}}$ (or $-f_{\mathrm{b}}$) is identified without any assumed parametric form, are solved in [3, 4, 5, 7]. However, those methods are not suitable for the application to sedimentation for different reasons, such as ill-posedness or regularity assumptions that are not satisfied here.

For a real suspension, the Kynch test [6] and the Diehl test [2], which are based on an initially homogenous suspension either filling the whole settling column or being initially located above clear liquid, respectively, provide data points that represent curved (convex or concave, respectively) suspension-supernate interfaces from which it is possible to reconstruct portions of the flux function to either side of the inflection point. Several functional forms can be employed to generate a provably convex or concave, twice differentiable accurate approximation of these data points via the solution of a constrained least-squares minimization problem. The resulting spline-like estimated trajectory can be converted into an explicit formula for the flux function. Thus, we do not assume an a priori parametric form of $f_{\mathrm{b}}$. It is proved that the inverse problem of flux identification solved this way has a unique solution. The problem of gluing together the portions of the flux function from the Kynch and Diehl tests is addressed. Examples involving synthetic data are presented. This presentation is based on [1].

**Keywords**: inverse problem, nonlinear hyperbolic partial differential equation, explicit representation, batch sedimentation

**Mathematics Subject Classifications (2000)**: 35L65, 35R30, 65M32

## References

[1] R. Bürger and S. Diehl. Convexity-preserving flux identification for scalar conservation laws modelling sedimentation. Preprint 2012-22, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción.
[2] S. Diehl. Estimation of the batch-settling flux function for an ideal suspension from only two experiments. *Chem. Eng. Sci.*, 62:4589–4601, 2007.
[3] E.M. Fernández-Berdaguer and G.B. Savioli. An inverse problem arising from the displacement of oil by water in porous media. *Applied Num. Math.*, 59(10):2452–2466, 2009.
[4] F. James and M. Sepúlveda. Convergence results for the flux identification in a scalar conservation law. *SIAM J. Control Optim.*, 37(3):869–891, 1999.
[5] H. Kang and K. Tanuma. Inverse problems for scalar conservation laws. *Inv. Problems*, 21:1047–1059, 2005.
[6] G.J. Kynch. A theory of sedimentation. *Trans. Faraday Soc.*, 48:166–176, 1952.
[7] G.B. Savioli and E.M. Fernández-Berdaguer. The estimation of oil water displacement functions. *Latin American Applied Res.*, 37(3):187–194, 2007.

CI$^2$MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile
*E-mail address*: rburger@ing-mat.udec.cl

Centre for Mathematical Sciences, Lund University, P.O. Box 118, S-221 00 Lund, Sweden
*E-mail address*: diehl@maths.lth.se