AN A POSTERIORI ERROR ESTIMATOR FOR THE ELECTROSTATICS PROBLEM WITH A CURRENT DIPOLE SOURCE

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Abstract. Electroencephalography is a non-invasive technique for detecting the brain activity from the measure of the electric potential on the head surface. In mathematical terms, it reduces to an inverse problem in which one wants to determine the source that has generated the electric field by measuring the boundary value of the electric potential [1]. Since the time-variation of the electric and magnetic fields are not relevant, the mathematical modeling of the corresponding forward problem leads to an electrostatics problem with a current dipole source. This is a singular problem, since the current dipole model involves first-order derivatives of a Dirac delta measure. This problem was recently proved to be well posed [3] and its solution was shown to lie in L^p for $1 \leq p < 3/2$.

We propose a residual-type a posteriori error estimator in L^p-norm ($1 \leq p < 3/2$) for the numerical approximation of the forward problem by means of standard piecewise linear continuous finite elements. With this end, we follow an approach similar to that in [2]. We prove that the estimator is reliable and efficient; namely, it yields global upper and local lower bounds for the corresponding norms of the error. We use this estimator to guide an adaptive procedure, which is experimentally shown to lead to an optimal order of convergence. Finally, we propose an strategy to solve the inverse problem based on this adaptive procedure, in order to improve the accuracy of the resulting algorithm. We report numerical results which allow assessing the efficiency of this approach.

Keywords: A posteriori error estimator, electrostatics problem, dipole source term.

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