A MULTISCALE METHOD FOR COMPRESSIBLE LIQUID-VAPOR FLOW WITH SURFACE TENSION

CHRISTOPH ZEILER

Abstract. We consider a compressible fluid in an open bounded domain $\Omega \subset \mathbb{R}^d$ with $d \in \{1,2,3\}$. It can appear either in a liquid or in a vapor phase. For any time $t > 0$ the domain Ω splits up into the union of two open domains $\Omega_{\text{vap}}(t)$, $\Omega_{\text{liq}}(t)$, which contain the two bulk phases, and a phase boundary $\Gamma(t)$ that separates the two bulks. In the two bulk phases we assume isothermal Euler equations

$$\begin{aligned} \rho_t + \operatorname{div}(\rho\,\boldsymbol{v}) &= 0, \\ (\rho\,\boldsymbol{v})_t + \operatorname{div}\left(\rho\,\boldsymbol{v}\otimes\boldsymbol{v} + p(\rho)\,\boldsymbol{I}\right) &= \boldsymbol{0}, \end{aligned} \tag{1}$$

for $t > 0$, $\boldsymbol{x} \in \Omega_{\text{vap}}(t) \cup \Omega_{\text{liq}}(t)$ and unknown density and velocity fields $\rho = \rho(\boldsymbol{x},t) > 0$ and $\boldsymbol{v} = \boldsymbol{v}(\boldsymbol{x},t) \in \mathbb{R}^d$. The given function $p = p(\rho)$ is a non-monotone equation of state, e.g., the Van-der-Waals pressure. The phase boundary is represented by the (dynamic) sharp interface $\Gamma(t) \subset \mathbb{R}^d$ at time $t > 0$. Let σ denote the speed of the phase boundary in normal direction $\boldsymbol{n}$. Across the interface the trace conditions

$$[\![\rho(\boldsymbol{v}\cdot\boldsymbol{n} - \sigma)]\!] = 0, \tag{2}$$

$$[\![\rho(\boldsymbol{v}\cdot\boldsymbol{n} - \sigma)\,\boldsymbol{v} + p(\rho)\,\boldsymbol{n}]\!] = (d-1)\gamma\kappa\,\boldsymbol{n}, \tag{3}$$

have to be satisfied. The surface tension coefficient $\gamma > 0$ is assumed to be constant and κ denotes the mean curvature of the phase boundary $\Gamma(t)$ associated with orientation given through the choice of the normal $\boldsymbol{n}$.

We will present a heterogeneous multiscale method, in the sense of [3], for this free boundary value problem, cf. [1]. On the microscale we consider the fluid dynamics at the phase boundary, more precisely around a segment of the phase boundary. The governing equations are (1), (2), and (3). The local view on the interface allows us to reduce microscale problems to Riemann type problems.

Macroscale grid cells are identified with the fluid either in the liquid or in the vapor phase. Then standard FV or DG schemes for (1) are applicable for single phase areas. More complicated is the coupling of the scales and the correct treatment of the phase boundary, which is not aligned to the grid. For communication of the fluid variables ρ and $\boldsymbol{v}$ between macro- and microscale we use a ghost fluid like approach, cf. [2]. The dynamics of the phase boundary is treated with an additional level set equation $\phi_t + \sigma\,|\nabla\phi| = 0$ for $t > 0$ and $\boldsymbol{x} \in \Omega$. In particular, we drive the level set function $\phi = \phi(\boldsymbol{x},t) \in \mathbb{R}$ with the interface speed σ, available from the microscale.

Multidimensional numerical examples will show how surface tension affects the behavior of bubbles respectively droplets of compressible fluids. We validate the procedure on stationary two phase solutions and estimate experimentally the order of convergence.

Keywords: heterogeneous multiscale method, liquid-vapor flow, surface tension

Mathematics Subject Classifications (2000): 76T10

References

[1] Felix Jaegle, Christian Rohde, and Christoph Zeiler. A multiscale method for compressible liquid-vapor flow with surface tension. accepted for publication in ESAIM Proceedings.

[2] Christian Merkle and Christian Rohde. The sharp-interface approach for fluids with phase change: Riemann problems and ghost fluid techniques. *M2AN Math. Model. Numer. Anal.*, 41(6):1089–1123, 2007.

[3] E Weinan, Bjorn Engquist, Xiantao Li, Weiqing Ren, and Eric Vanden-Eijnden. Heterogeneous multiscale methods: a review. *Commun. Comput. Phys.*, 2(3):367–450, 2007.

University of Stuttgart
E-mail address: christoph.zeiler@mathematik.uni-stuttgart.de