BEST N-TERM CHEBYSHEV APPROXIMATIONS OF MULTIVARIATE GAUSSIANS

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ABSTRACT. Many problems in statistical learning depend on computing the discrete Gauss transform efficiently. The task is to evaluate the potentials

$$\Phi(x_i) = \sum_{j=1}^{N} \exp\left(-\frac{|x_i - y_j|^2}{\delta}\right) w_j$$

for given sources $y_j \in [0,1]^d$, targets $x_i \in [0,1]^d$, weights w_j and variance δ.

The fast Gauss transform is a well known algorithm to overcome the $O(N^2)$ complexity of the direct evaluation. This algorithm subdivides the data set into smaller clusters and approximates interactions between two clusters using separation of variables. For larger values of the dimension d, the success of the fast algorithm depends strongly on how Gaussian is expanded. The original Hermite expansion [1] and the exponential expansion [2] approximate well only within relatively small clusters and scale exponentially in d. The multivariate Chebyshev series [4] has better global approximation properties, but still suffers from the curse of dimensionality.

Many realistic data sets have low-dimensional features which can be exploited by allowing for clusters with arbitrary orientations and large aspect ratios, see, e.g., [5, 3]. In this case the Gaussian can be expanded more efficiently in the coordinates of the principal axes. This talk will present an algorithm that will generate an optimal N-term multivariate Chebyshev series approximation, exploiting tree-structures and special properties of the one-dimensional Chebyshev series. We will also discuss algorithms that preserve the tensor product form of the Gaussian to rapidly compute Gaussian sums.

Keywords: Gauss transform, Curse of dimension, N-term approximation.

Mathematics Subject Classifications (2000): 65D15, 42B99

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