ON SPACE-TIME APPROACHES IN REDUCED BASIS METHODS FOR TIME-PERIODIC PDES

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Abstract. Reduced basis methods for parameterized partial differential equations rely on rigorous and sharp a posteriori error bounds. In many time-dependent contexts, classical reduced basis schemes [1] face the problem that both error estimates as well as the required bounds for stability constants grow exponentially in time. Recent advances show, however, that space-time approaches are able to overcome this challenge and yield sharp and stable estimates even for long-time integration [2, 5].

We concentrate on space-time reduced basis formulations in the context of parameterized time-periodic parabolic PDEs [4]. Such equations arise for example in the design or steering of rotators or propellers. Their computational cost is even larger than that of common initial value problems, rendering them particularly eligible for treatment with model reduction methods.

We derive rigorous a posteriori error estimates for such problems and present appropriate discretization methods. Due to time-periodicity, finite element discretizations like in [2, 5] result in a fully coupled system. As an alternative, we discuss adaptive space-time wavelet methods [3].

Unlike classical time-stepping fixed-point methods, this approach allows a simultaneous reduction in both temporal and spatial dimensions, resulting in reduced systems with very short solution runtimes. This and the effectivity of the space-time error estimates is demonstrated for a convection-diffusion-reaction example.

Keywords: space-time formulation, reduced basis, a posteriori estimation, time-periodicity, inf-sup stability

Mathematics Subject Classifications (2000): 35K99, 65M15, 65M60

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