APPROXIMATION AND STABILITY PROPERTIES OF MULTISCALE DISCRETISATION SCHEMES

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Abstract. This talk will focus on weak approximation and stability properties of various standard and multiscale discretization schemes for 2nd order elliptic boundary value problems with large coefficient jumps that are not resolved by the underlying grid. The analysis uses new results on uniform weighted Poincaré inequalities from [2] for locally quasi-monotone coefficients and an abstract Bramble-Hilbert lemma in [3] for more general coefficients. The main area of application for these results is in the context of multilevel iterative methods [3, 4] to establish uniform convergence. However, it also allows to establish approximation results for best approximants in standard and weighted L_2-norms. Using an equivalence result and two-sided bounds established in [1, 5], approximation properties can be directly linked to the convergence of two-level iterative schemes which in turn allows to give necessary conditions for convergence.

Keywords: high contrast coefficients, multiscale finite elements, energy minimizing bases, spectral coarse spaces, coefficient robustness

Mathematics Subject Classifications (2000): 65C30, 65N55, 65N12

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