

ON THE ELLIPTICITY OF THE JOHNSON–NÉDÉLEC COUPLING OF FINITE AND BOUNDARY ELEMENT METHODS FOR BOUNDARY VALUE PROBLEMS

GÜNTHER OF AND OLAF STEINBACH

ABSTRACT. We present the extension of recent results on the stability of the Johnson–Nédélec coupling [1, 2] of finite and boundary element methods to the case of boundary value problems.

In [3, 4, 5] the case of a free–space transmission problem was considered, and sufficient and necessary conditions are stated which ensure the ellipticity of the bilinear form for the coupled problem. The proof was based on the relation of the energies which are related to both the interior and exterior problem.

When considering boundary value problems for both interior and exterior problems, additional estimates to bound the energy for the solutions of related subproblems are required. Moreover, several techniques for the stabilization of the coupled formulations are analyzed. Applications involve boundary value problems with either hard or soft inclusions, exterior boundary value problems, and macro–element techniques.

Keywords: Johnson–Nédélec coupling, finite element method, boundary element method

Mathematics Subject Classifications (2000): 65N12, 65N30, 65N38

REFERENCES

- [1] F. Brezzi, C. Johnson. On the coupling of boundary integral and finite element methods. *Calcolo* 16:189–201, 1979.
- [2] C. Johnson, J.–C. Nédélec. On the coupling of boundary integral and finite element methods. *Math. Comp.* 35:1063–1079, 1980.
- [3] G. Of, O. Steinbach. Is the one–equation coupling of finite and boundary element methods always stable? Berichte aus dem Institut für Numerische Mathematik, Report 2011/6, Graz University of Technology, 2011.
- [4] F.–J. Sayas. The validity of Johnson–Nédélec’s BEM–FEM coupling on polygonal interfaces. *SIAM J. Numer. Anal.* 47 (2009) 3451–3463.
- [5] O. Steinbach. A note on the stable one–equation coupling of finite and boundary elements. *SIAM J. Numer. Anal.* 49:1521–1531, 2011.

INSTITUTE OF COMPUTATIONAL MATHEMATICS, GRAZ UNIVERSITY OF TECHNOLOGY, STEYRERGASSE 30,
8010 GRAZ, AUSTRIA

E-mail address: of@tugraz.at

INSTITUTE OF COMPUTATIONAL MATHEMATICS, GRAZ UNIVERSITY OF TECHNOLOGY, STEYRERGASSE 30,
8010 GRAZ, AUSTRIA

E-mail address: o.steinbach@tugraz.at