INTERFACE PERTURBATION ANALYSIS FOR COUPLED SYSTEMS OF MULTIPLE SCALE FLOW IN MIXED FORMULATION

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Abstract. Modeling the phenomenon of fluid flow in layered media naturally presents a multiple scale flow problem of regions of slow velocity of order $\mathcal{O}(1)$ in a rock matrix and fast flow of order $\mathcal{O}(1/\epsilon)$ in the fissures together with fluid exchange conditions through and interface Γ. However, finding the numerical solutions of such systems introduces implicitly a perturbation of the interface whenever the original one is a curved, non-polynomial surface. Since the interface couples regions of different scale flow such perturbation can introduce and important error which we address in this work. The regions defined by the original and perturbed interfaces as well as the disagreement in the order of velocity hint a triple coupled system in mixed variational formulation model to introduce the necessary number of degrees of freedom on the test functions and to make both solutions comparable *i.e.* belonging to the same space. We state conditions on the forcing terms as well as the geometric perturbation in order to conclude continuity statements with respect to the perturbation of the interface for fixed data. For numerical purposes we give conditions of stability and due to the nature of the continuity estimates we give a-posteriori perturbation/error estimates *i.e.* in terms of the approximate solution. Also, the continuity estimates suggest numerical experimentation as the most feasible approach to the convergence rate problem. Finally, we show that the question of continuity with respect to geometric perturbations of the interface in direct variational formulation, even in the simplest case, is highly non-linear for dimensions higher than one and therefore not a recommendable framework.

Keywords: fissured media, multiscale flow, geometric perturbation, PDE coupled systems, mixed variational formulation, numerical stability, a-posteriori estimates.

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