STABILIZED HYBRID AND DG METHODS FOR SECOND ORDER ELLIPTIC PROBLEMS

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Abstract. The natural connection between Discontinuous Galerkin (DG) [1] formulations and hybrid methods [2] have been successfully exploited to derive new finite element methods with improved stability and reduced computational cost keeping the robustness and flexibility of DG methods [3, 4, 5]. Here we propose a class of stabilized dual hybrid mixed finite element formulations for second order elliptic problems considering as a model problem Darcy flow in porous media in its velocity and pressure formulation. To improve stability we add least squares residuals of the governing equations and the interface conditions as in [6, 7] and [8], for example. A global system is assembled involving only the degrees of freedom associated with the Lagrange multiplier identified with the trace of the pressure field on the element boundaries. Numerical results are presented using equal order polynomial approximations for all fields considering continuous or discontinuous interpolations for the Lagrange multiplier. A great reduction on computational cost is obtained with continuous interpolations for the multiplier, and higher order polynomial approximations are shown to be much more accurate than lower lower order polynomial approximations with the same number of global degrees of freedom.

Keywords: Discontinuous Galerkin, hybrid method, mixed method, Darcy flow, stabilization, hybridization

Mathematics Subject Classifications (2000):

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