A ROBUST LAYER ADAPTED DIFFERENCE METHOD FOR SINGULARLY PERTURBED TWO PARAMETER PARABOLIC PROBLEMS

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Abstract. We consider the following two parameter singularly perturbed parabolic boundary value problem on $\Omega = (0,1) \times (0,T]$

$$\begin{aligned}
\mathcal{L}(u) \equiv \epsilon\, u_{xx} + \mu\, a\, u_x - b\, u - d\, u_t &= f(x,t) \quad \text{in } (x,t) \in \Omega, \\
u &= u_0(x) \quad \text{on } \Gamma_b = \{(x,0) \,|\, 0 \le x \le 1\} \\
u &= s_0(t) \quad \text{on } \Gamma_l = \{(0,t) \,|\, 0 \le t \le T\} \\
u &= s_1(t) \quad \text{on } \Gamma_r = \{(1,t) \,|\, 0 \le t \le T\}
\end{aligned} \tag{1}$$

where $0 < \epsilon \ll 1$ and $0 < \mu \ll 1$ are two known small parameters. The coefficients a, b, d and f are sufficiently regular on $\bar{\Omega}$, and $0 < \alpha \le a(x,t)$, $0 < \beta \le b(x,t)$, $0 < \delta \le d(x,t)$. We also assume sufficient regularity of initial-boundary data on $\Gamma = \Gamma_b \cup \Gamma_l \cup \Gamma_r$ and compatibility at the corners, i.e. $u_0(0) = s_0(0)$, $u_0(1) = s_1(0)$, so that unique solution exists and is sufficiently regular for our purpose.

The solution of these problems have multiscale character, having rapid variations in the solution in regions close to both the left and the right lateral boundaries depending upon the size of the parameters ϵ and μ. For $\mu = 1$, the exponential boundary layer of width O(ϵ) in the neighborhood of left lateral boundary appears, whereas for $\mu = 0$, parabolic boundary layer of width O ($\sqrt{\epsilon}$) appear on both the left and right lateral boundaries. We discuss the treatment of the problem for the possible classes of the parameter space for $0 < \mu < 1$ with $\epsilon/\mu^2 \to 0$ and $\mu^2/\epsilon \to 0$ as $\mu \to 0$ and $\epsilon \to 0$ respectively.

A finite difference method for the above time-dependent singularly perturbed convection-diffusion-reaction problem involving two small parameter in one space dimension is considered. We use classical implicit Euler method for time discretization and upwind scheme on Shishkin-Bakhvalov mesh for spatial discretization. The method is analyzed for convergence and is shown to be uniform with respect to both the perturbation parameters. The use of Shishkin-Bakhvalov mesh gives the first order convergence unlike Shishkin mesh where convergence is deteriorated due to the presence of the logarithmic factor. Numerical results are presented to validate the theoretical estimates obtained..

Keywords: singular perturbations; two parameters; Shishkin-Bakhvalov mesh; upwind method; parabolic problem; uniform convergence

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