# NONLINEAR DIFFUSION FILTERING OF DATA ON THE EARTH'S SURFACE

RÓBERT ČUNDERLÍK AND KAROL MIKULA

Abstract. We present new models for data filtering based on numerical solutions of linear and nonlinear diffusion equations on closed surfaces. To that goal we introduce a surface finite-volume method (SFVM) that approximate numerically parabolic PDEs on closed surfaces, namely on the Earth's surface [2]. The Earth is approximated by a polyhedral surface created by planar triangles and we construct a dual co-volume grid. On the co-volumes we define a weak formulation of the problem by applying Green's theorem to the Laplace-Beltrami operator. Then SFVM is applied to discretize the weak formulation. The weak forms of elliptic operators are expressed through surface gradients [3]. In our numerical scheme we use a piece-wise linear approximation of a solution in space and the backward Euler time discretization. Furthermore, we extend a linear diffusion on surface to the regularized surface Perona-Malik model. It represents a nonlinear diffusion equation, which at the same time reduces a noise and preserves main edges and other details important for a correct interpretation of the real data [4, 1].

We present four numerical experiments. The first one has an illustrative character showing how an additive noise is filtered out from an artificial function defined on a sphere. Other three examples deal with the real geodetic data on or above the Earth's surface, e.g. we filter real measurements of the satellite mission GOCE that monitors the Earth's gravity field or we reduce a stripping noise from the mean dynamic topography at oceans observed by satellite altimetry. In all experiments we focus on a comparison of the results obtained by both the linear and nonlinear models presenting advantages of the nonlinear diffusion.

**Keywords**: data filtering on a closed surface; linear and nonlinear diffusion equations; surface finite volume method; surface gradients; the regularized surface Perona-Malik model

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*Faculty of Civil Engineering*, Slovak University of Technology, *Radlinskeho 11*, 813 68 Bratislava, Slovakia

*E-mail address*: cunderli@svf.stuba.sk

*Faculty of Civil Engineering*, Slovak University of Technology, *Radlinskeho 11*, 813 68 Bratislava, Slovakia

*E-mail address*: mikula@math.sk