USING POTENTIALS IN LINEAR ELASTODYNAMICS : A CHALLENGE FOR FINITE ELEMENT METHODS ?

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Abstract. The use of scalar and vector potentials is a very basic tool, that can be found in any text book, for computing analytical solutions of linear isotropic elastodynamics equations in homogeneous media. Curiously, this technique appears to very rarely used for numerical computations despite of the fact that there are many situations where this could be useful, in particular to treat a domain of propagation made of several isotropic homogeneous media separated by interfaces. The idea on using potentials, which permit to separate the propagation of P and S waves, could be more specifically interesting in the case of media in which the shear waves propagate much more slowly than pressure waves (like biological tissues for instance) : in such a case one could think to use different computational meshes and / or time steps for computing P waves and S waves, which would not be possible when working directly with displacement fields. In this work, we investigate the possibility of combining the decomposition of the displacement fields in potentials with the use of finite elements for the space discretization. The major (and somewhat surprising) difficulty lies in the treatment of the coupling between the two types of waves, which occurs at physical boundaries or interfaces via boundary and transmission conditions. This is not obvious even for the treatment of rigid boundaries (Dirichlet condition for the displacement field). In this case, we shall present a solution for which the theory is now complete and which gives satisfactory numerical results. The case of free boundaries (generalized Neumann conditions) is still open at the time we write this abstract. We shall present the ideas that we tried and the difficulties that we encounter. The work is still in progress. We conjecture that if we would be able to treat both Neumann and Dirichlet conditions, the transmission condition should follow easily.

Keywords: elastodynamics, potentials, decomposition, finite elements, energy

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References

[1] P. Joly, Finite element methods with continuous displacement. *Chapman & Hall/CRC, Boca Raton, FL*, pp. 267-329, 2008.

[2] P. Joly, The mathematical model for elastic wave propagation. *Chapman & Hall/CRC, Boca Raton, FL*, pp 247-266, 2008.

[3] J. Diaz and P. Joly, Robust high order non-conforming finite element formulation for time domain fluid-structure interaction. In : *Journal of Computational Acoustics. An IMACS Journal*, Vol 13 (3), pp 403-431, 2005.

[4] P. Joly, Variational Methods for Time Dependant Wave Propagation. In :*Topics in computational wave propagation : Direct and Inverse Problems. Computational Methods in Wave Propagation*, Ainsworth, M and Davies, P and Duncan, D and Martin, P and Rynne, B (eds), pp 201-264, Springer Verlag, oct 2003.

[5] P. Monk, Finite element methods for Maxwell's equations. Oxford science publications, 2003.

[6] G. Cohen, Higher-order numerical methods for transient wave equations. Springer-Verlag, Berlin, 2001.

[7] J. Achenbach, Wave Propagation in Elastic Solids, North Holland, Jan 1975.

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