FINITE ELEMENT APPROXIMATION OF MAXWELL'S EIGENVALUES ON GENERAL MESHES OF QUADRILATERALS AND CUBES

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Abstract. The finite element approximation of Maxwell's eigenvalues is a well-studied problem and general sufficient and necessary conditions for its convergence analysis are available in the literature. When the elements of the mesh are constructed as the tensor product of intervals (like for quadrilaterals or cubes), it is well-known that standard approximation properties of finite element spaces can degrade if the elements get distorted. We review this topic and propose some cures.

In particular, we recall that the approximation properties of finite element spaces depend on the reference space and on the transformations used for their definition. Necessary and sufficient conditions for optimal approximation are available when the transformation is the composition with the inverse of the mapping from the reference to the actual element; in this case approximation order $r+1$ in L^2 and r in H^1 is achieved when the reference space contains all polynomial functions of separate degree r in each variable. When different transformations are considered, the situation is more complicated and only partial results are known. In particular, Raviart–Thomas elements do not achieve optimal convergence in $H(\mathrm{div})$ on quadrilaterals and not even in L^2 on hexahedrons when the Piola transform is used for the construction of the function spaces on the actual elements. Similar results occur for Nédélec spaces (*edge elements*) which are commonly used for the approximation of Maxwell's eigevalues.

The framework of Finite Element Exterior Calculus makes it possible to embed this problem in a much more general picture and to analyze it in a unified way. We discuss the abstract construction of tensor product of complexes of differential forms. This allows in a natural way the definition of shape functions and degrees of freedom for finite element differential forms of order k on cubes in n dimensions. In particular, the case of edge elements correspond to differential forms of first degree. This construction can be extended via the pullback transformation to curvilinear cubic elements, obtained as images of a reference n-cube. In this context we study the approximation properties of the resulting finite element spaces. When the maps from the reference cube are affine, the approximation rate depends only on the degree of polynomials contained in the reference space; in the more general case, when the maps are multilinear, a degradation in the approximation rate is observed, the loss being more severe for higher degree differential forms.

Keywords: Maxwell's eigenvalues, distorted elements, finite element exterior calculus.

Mathematics Subject Classifications (2000): 65N25, 65N30

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