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## Numerical simulation of thermally convective viscoplastic fluids by semismooth second order type methods\*

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### Abstract

This talk is focused on the numerical solution of thermally convective viscoplastic fluids with yield stress. Following [4], a Bousinesq approximation of the convection effect is considered. The resulting coupled model is then regularized by means of a local regularization technique (see [2, 1]). We discuss a discretization in space by using a finite difference approach based on MAC scheme on staggered grids (see [3]). After space discretization, a second order BDF method is used for the time discretization of the regularized problem, leading, in each time iteration, to a nonsmooth system of equations, which is amenable to be solved by generalized Newton methods (see [2]). A semismooth Newton algorithm with a modified Jacobian is constructed for the solution of the discrete systems. Finally, we present a detailed computational experiment that exhibits the main properties of the numerical approach.

**Key words:** Thermal convection, viscoplastic fluids, yield stress, BDF methods, Semismooth Newton methods.

**Mathematics subject classifications (1991):** 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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