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On the use of high-order schemes for seismic imaging*

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Abstract

Seismic imaging involves the propagation of waves generated by artificial sources to produce maps of the subsurface. Images are reconstructed from measurements by sensors recording signals from the reflected waves which contain two types of information. The first one contains the kinematics of the propagation phenomenon and its determination is crucial to pinpoint the different reflectors. Then, computing the kinematics accurately provides an efficient tool capable of drawing the edges of each structural region composing the subsurface. The second one reproduces the dynamics of the propagation medium which is used to determine the material properties of each geological layer constituting the subsurface. From a mathematical point of view, both issues belong to the class of inverse problems but they do not use the solutions of wave equations in the same way. This work focuses on numerical methods which are used to deliver information on the kinematics related to the propagation of waves in heterogeneous media. Applied mathematicians use to concentrate efforts on the accuracy of the numerical solutions by developing more and more advanced numerical methods. But, in the context of seismic imaging, this is not the only drawback. Indeed, seismic imaging delivers images of the subsurface from the cross-correlation of a collection of solutions of wave equations. Then, for a given accuracy, conventional numerical methods quickly reach their limitations because the production of images require to store away a huge number of snapshots. Hence, an advanced numerical method for seismic imaging must deliver accurate solutions to wave equations with an optimal use of the computer memory. Numerical methods such as finite element schemes are well-known to capture accurately the properties of wave propagation in highly heterogeneous media. The price to pay is high computational burdens which can be partially balanced by the use of parallel computing. Numerical methods for seismic imaging must thus involve computations which can be well-distributed to the processors. Following this point of view, Discontinuous Galerkin (DG) approximations are very attractive since they are suitable for parallel computations and provide very flexible approximation tools particularly adapted to reproduce wave propagation in highly contrasted media. DG methods are moreover *hp*-adaptive and can be applied on general meshes composed of tetrahedra or hexahedra.

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They are therefore able to limit the occupation of memory as they allow to optimize the number of degrees of freedom due to the combination of different orders of approximation determined by regional physical characteristics of the propagation medium. DG methods have also the noteworthy advantage to produce block-diagonal mass matrices. The discretization procedure can thus be completed with an explicit time approximation which increases the capability of the numerical method to stay within the limits of the memory space. The Leap-Frog scheme is the most widely used explicit time discretization. It is of order 2 only and therefore, it does not let to take fully advantage of high order space discretization. High-order time schemes have then been developed and DG-ADER methods [2] emerged as extension of the Modified Equation Technique [3, 4]. Regarding the memory use, DG-ADER schemes are relevant because they are single step time integration procedures. Thus, they only require to store the solution at the previous time step. Nevertheless, memory limits can be reached in particular when solving 3D wave equations because they require to introduce auxiliary unknowns. A time scheme which requires less memory than DG-ADER methods for a given level of accuracy is thus mandatory for seismic imaging. The purpose of this work is to construct a new higher order time scheme for wave equations by exploiting the capability of DG functions to easily approximate high order differential operators. A Modified Equation Technique can thus be applied, but by applying the time integration first. High order differential operators are then introduced but their discretization by a DG method is straightforward. The resulting single-step time scheme is of arbitrary order and demonstrates a high level of accuracy while creating acceptable computational costs. In particular, for a given accuracy, the new scheme allows for using coarser meshes than with DG-ADER methods. The storage and the computational times are thus considerably reduced. These concluding remarks are obtained by validating the time scheme with the DG formulation proposed in [1] on toy problems. The validation for realistic configurations for seismic imaging sets next the difficult question of introducing seismic sources. This issue suggests to modify the DG scheme proposed in [1] by adding a penalization term which may hamper the performances of explicit time discretization schemes. We illustrate this point by performing numerical experiments in realistic configurations.

Key words: Discontinuous Galerkin approximations, higher order explicit time schemes, elastodynamic equation, seismic imaging

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 35J05

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